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1 (Sem-4/FYUGP) STA 42 MN/(A)

2026

STATISTICS

(Minor)

SET-A

Paper : STA4400604MN

**(Sampling Distribution and
Test of Significance)**

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate
full marks for the questions.*

SET-A

1. Answer the following questions :

(answer **all eight** questions)

(a) The standard error of sample mean of a sample of size 'n' from a Normal Population with mean ' μ ' and variance ' σ^2 ' is _____. (Fill in the blank)

1

(b) The m.g.f. of a random variable X following chi-square distribution with n degrees of freedom is _____.

(Fill in the blank)

1

(c) For validity of χ^2 -test, no cell frequency should be less than 5.

(State True **or** False)

1

(d) t -distribution tends to normal distribution as sample size tends to infinity.

(State True **or** False)

1

(e) Large sample theory is based on _____ distribution. (Fill in the blank)

1

(f) If $F = \frac{\chi_1^2/n_1}{\chi_2^2/n_2}$ follows F distribution with

n_1 and n_2 degrees of freedom, then $\frac{1}{F}$

follows _____ with n_2 and n_1 degrees of freedom. (Fill in the blank)

1

(g) Give the 95% Confidence Interval for μ (specified) based on Normal distribution when n is the sample size and σ (specified) is the population standard deviation.

1

(h) The t -statistic for testing the significance of difference of means is _____.

(Fill in the blank)

1

2. Answer **any six** of the following :

(a) Define Parametric and Statistic. 2

(b) Define Sampling Distribution and Standard Error. 2

(c) Find the parameters sample mean when sample mean, $\bar{x}$ is known to follow Normal distribution. 2

(d) A normal distribution has mean 0.5 and standard deviation 2.5. Find the probability that the mean of a random sample of size 16 from the population is positive. 2

(e) State the Additive property of chi-square variate. 2

(f) Show that students 't' may be regarded as a particular case of Fisher's 't'. 2

(g) What are the assumptions underlying student's t-test? 2

(h) Let (X_1, X_2) be a random sample from $N(0,1)$, then what is the distribution of

(i) $Z_1 = \frac{X_1^2}{X_2^2}$

(ii) $Z_2 = X_1 + X_2$, when X_1 and X_2 are independent

2

(i) Write *two* applications of F-distribution.

2

(j) A random sample of 27 pairs of observations from a normal population gave a correlation co-efficient of 0.6. Is this significant of correlation in the population? (Given $t_{0.05}$ for 25 d.f is 2.06) 2

3. Answer **any four** of the following :

(a) Derive the relation between F-distribution and chi-square distribution. 5

(b) Briefly explain : 5

(i) Z-test for single proportion

(ii) Z-test for difference of means

(c) Obtain 100 $(1 - \alpha)\%$ Confidence Interval for the parameter σ^2 of the normal distribution having probability density function

$$f(x; \theta; \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - \theta}{\sigma} \right)^2 \right]$$

$$-\infty < x < \infty$$

$$-\infty < \mu < \infty$$

$$\sigma^2 > 0$$

when θ is known and equal to μ .

5

- (d) Define Confidence Interval and Confidence limit. Also give the procedure to find the lower and upper confidence limits for a parameter. 5
- (e) Write the conditions for validity of chi-square test. 5
- (f) Test the hypothesis that $\sigma = 10$, given that the sample standard deviation $s = 15$ for a random sample of size 50 from a normal population. 5
- (g) The p.d.f. of a random variable X is given by : 5

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2}, & 0 < x < \infty \\ 0, & \text{elsewhere,} \end{cases}$$

find mean, variance and moment generating function of X .

- (h) Prove that the statistic F satisfies $F_{\alpha}(n_1, n_2) \times F_{1-\alpha}(n_2, n_1) = 1$ where $F_{\alpha}(.1.)$ and $F_{(1-\alpha)}(.1.)$ are the critical values of F at $\alpha\%$ and $(1-\alpha)\%$ level of significance. 5

4. Answer **any four** of the following :

- (a) The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study the following information was obtained.

Days	:	Mon	Tue	Wed	Thurs	Fri	Sat
No. of parts demanded	:	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week.

(Given $\chi^2_{0.05}(5) = 11.07$) 5

- (b) Describe the nature of probability density curve with different degrees of freedom for Snedecor's F -distribution. 5
- (c) A random sample of 10 boys had the following IQ's :
70, 120, 110, 101, 88, 83, 95, 98, 107, 100
Do these data support the assumption of a population mean I.Q of 100 ? 5
- (d) Describe t -test for testing the significance of an observed sample correlation co-efficient. 5

- (e) A die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one. Also find the limits between which the probability of a throw of 3 or 4 lies. 5
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1 (Sem-4/FYUGP) STA 43 MN/(A)

2026

STATISTICS

(Minor)

SET-A

Paper : STA4400704MN

**(Estimation Theory and Non-Parametric
Methods)**

Full Marks : 45

Time : 2 hours

*The figures in the margin indicate
full marks for the questions.*

SET-A

1. Answer the following questions : $1 \times 5 = 5$

(a) If an estimator T_n of population parameter θ converges in probability to θ as n tends to infinity is said to be

- (i) Sufficient
- (ii) Efficient
- (iii) Consistent
- (iv) Unbiased

(b) A maximum likelihood estimator may be consistent but not necessarily _____.

(c) Kolmogorov-Smirnov test is

- (i) One left-sided test
- (ii) One right-sided test
- (iii) Two-sided test
- (iv) All the above

(d) To test the randomness of a sample, the appropriate test is

- (i) Sign test
- (ii) Median test
- (iii) Run test
- (iv) All of the above

(e) For the mean square error to be minimum, bias should be _____.

2. Answer **any five** of the following questions : $2 \times 5 = 10$

(a) What assumptions are generally made for a non-parametric test?

(b) If T is an unbiased estimator for θ , show that T^2 is a biased estimator for θ^2 .

(c) Write the basic principle of test of one sample run test.

(d) Write the *two* properties of Maximum Likelihood Estimators.

(e) Write down the limitations of non-parametric test.

(f) Define consistent estimator.

(g) Define one-sided confidence interval.

(h) Show that unbiased estimators does not always exist.

- (i) What is the difference between point estimation and interval estimation?
- (j) Write the properties of the estimates obtained by the method of moments.

3. Answer **any four** of the following questions :
5×4=20

- (a) Write a short note on the method of moments.
- (b) Prove that the invariance property of consistent estimators.
- (c) Let $x_1, x_2, \dots, x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimator for μ and σ^2 .
- (d) Let $x_1, x_2, \dots, x_n$ be a random sample from of size n from a Poisson population with parameter λ . Find the unbiased estimators of λ and λ^2 . Also find the variance of the estimator of λ .

- (e) Let $x_1, x_2, \dots, x_n$ be a random sample of n observations from $N(\theta, \theta)$. Find an unbiased and efficient estimator of θ .

- (f) If $x_1, x_2, \dots, x_n$ are n observations of a random sample from a population whose probability function is

$$P(x) = \begin{cases} \frac{1}{b-a} & , \quad a < x < b \\ 0 & , \quad \text{otherwise} \end{cases}$$

Let ' a ' is known. Find the consistent estimator of b .

- (g) Write a short note on test of randomness.
- (h) Explain the general procedure for parametric interval estimation on the basis of sample drawn from continuous distribution.

4. Answer **any one** of the following questions :
10×1=10

- (a) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ . Consider the following estimators to estimate μ , where

$$T_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5},$$

$$T_2 = \frac{X_1 + X_2}{2} + X_3 \text{ and } T_3 = \frac{2X_1 + X_2 + \lambda X_3}{3}$$

- (i) Are T_1 and T_2 unbiased estimator ?
(ii) Find λ such that T_3 is an unbiased estimator of μ .
(iii) State giving reasons, the estimator which is best among T_1 , T_2 and T_3 .
- (b) Describe briefly Kolmogorov-Smirnov test of goodness of fit in case of one sample.
- (c) In a random sampling from normal population $N(\mu, \sigma^2)$ find the maximum likelihood estimators for
(i) μ when σ^2 is known
(ii) σ^2 when μ is known

- (d) Let $x_1, x_2, \dots, x_n$ be a random sample of size n from a population having p.d.f.

$$f(x) = \begin{cases} \frac{2}{\theta^2} (\theta - x) & , \quad 0 \leq x \leq \theta \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find unbiased estimators of θ and θ^2 .
Also find the variance of estimator of θ .