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1 (Sem-4/FYUGP) STA 41 MJ

2026

STATISTICS

(Major)

Paper : STA4400104MJ

(Probability-2 and Probability Distributions-2)

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate
full marks for the questions.*

1. Answer the following questions as directed :

$$1 \times 8 = 8$$

(a) If X is a random variable having probability function $f(x)$, then the function $\sum e^{itx} f(x)$, for ' i ' be an imaginary unit, is known as

(i) Moment generating function

(ii) Probability distribution function

(iii) Characteristic function

(iv) None of the above

(Choose the correct option)

(b) If the mean of a gamma distribution is 3, what is its variance?

(c) The sum of independent beta variates is also a beta variate.

(State True or False)

(d) The variance of a normal distribution is 9, find the value of quartile deviation.

(e) Characteristic function of a random variable always exists.

(State True or False)

(f) If X is a continuous random variable with mean ' μ ' and variance ' σ^2 ', then for any positive number ' k '

$$P\{|X - \mu| \geq k\sigma\} < \frac{1}{k^2}$$

is known as

(i) Liapounoff's inequality

(ii) Markov's inequality

(iii) Chebyshev's inequality

(iv) None of the above

(Choose the correct option)

(g) Order statistics depend only on the order of the values, not the values themselves. *(State True or False)*

(h) The WLLN states that the sample mean converged in _____ to the theoretical mean as sample size ' n ' increases indefinitely. *(Fill in the blank)*

2. Answer **any six** questions of the following :
 $2 \times 6 = 12$

(a) Define joint probability mass function and joint probability density function.

(b) Can $P(x) = \frac{2}{1+x}$ be the p.g.f. of a random variable X ? Give reason.

- (c) What is order statistics? Mention *one* use of it.
- (d) State Levy-Lindeberg theorem.
- (e) Write the p.d.f. of Weibull distribution and state its mean.
- (f) Under what condition negative binomial distribution may be regarded as the generalization of geometric distribution and how?
- (g) If X and Y are random variables then prove that
- $$E[X] = E[E(X/Y)]$$
- (h) Write the conditions for the existence of weak laws of large numbers (WLLN).
- (i) If X follows exponential distribution with parameter 5. What are the value of the mean and variance of the distribution?
- (j) A random variable ' X ' has mean 100 and variance 16. Using Chebyshev's inequality find the upper bound for $P(|X - 100| \geq 8)$.

3. Answer **any four** of the following questions :
5×4=20

- (a) Two ideal dice are thrown. If X_1 be the score on the first die, X_2 be the score on 2nd die and Y be the maximum of X_1 and X_2 the find
- (i) Joint distribution of Y on X_1
- (ii) $E[Y]$
- (b) State and prove the additive property of Gamma distribution.
- (c) If the random variable ' X ' assumes the value ' r ' with probability law
- $$P(X = r) = q^{r-1}p, r = 1, 2, 3, \dots$$
- Find the m.g.f. of X and hence its mean and variance.
- (d) Define Exponential distribution. Find its mean and variance.
- (e) Derive the distribution of r th order statistics in taking a random sample of size ' n ' from a continuous distribution.
- (f) Define Hyper Geometric distribution and find its mean and variance.

(g) The p.d.f. of a random variable X is

$$f(x) = \frac{1}{\beta(m, n)} x^{m-1} (1-x)^{n-1}; 0 < x < 1.$$

Find the distribution of $Y = \frac{1}{X}$.

(h) Define negative binomial distribution. Obtain its m.g.f. and c.g.f.

4. Answer **any two** questions from the following: $10 \times 2 = 20$

(a) (i) What is Geometric distribution? Explain its lack of memory property. 5

(ii) If X follows standard Cauchy distribution, show that X^2 follows

$$\beta_2\left(\frac{1}{2}, \frac{1}{2}\right). \quad 5$$

(b) (i) State and prove Chebyshev's inequality. 7

(ii) If X is the number obtained in a throw of unbiased die, then using Chebyshev's inequality show that

$$P\{|X - \mu| \geq 2.5\} \leq 0.4667$$

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(c) State and prove the central limit theorem.

(d) The joint p.d.f. of the random variables X and Y is

$$f(x, y) = \begin{cases} \frac{2}{3}(x+2y) & ; \text{ for } 0 < x < 1, 0 < y < 1 \\ 0 & ; \text{ otherwise} \end{cases}$$

$$7+3=10$$

Find the conditional density of X given $Y=y$ and use it to evaluate

$$P\left(x \leq \frac{1}{2} / Y \leq \frac{1}{2}\right)$$

(e) (i) Define characteristics function and state its important properties. Also state the uniqueness theorem of characteristic function. 5

(ii) Define Lognormal distribution. State its properties and uses. 5

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1 (Sem–4/FYUGP) STA 42 MJ

2026

STATISTICS

(Major)

Paper : STA4400204MJ

(Mathematical Methods)

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate
full marks for the questions.*

1. Answer the following questions as directed :
1×8=8

(a) For positive integers n and m

(i) $\Delta^n 0^n = n!$ and $\Delta^m 0^n = 0, n > m$

(ii) $\Delta^n 0^n = 0$ and $\Delta^m 0^n = 0, n < m$

(iii) $\Delta^n 0^n = n!$ and $\Delta^m 0^n = 0, n < m$

(iv) None of the above

(Choose the correct option)

(b) A necessary condition for convergence of an infinite series $\sum U_n$ is that

$\lim_{n \rightarrow \infty} U_n = \underline{\hspace{2cm}}$. (Fill in the blank)

(c) According to L Hospital's rule for indeterminate form $\frac{0}{0}$, under certain conditions imposed upon the function

$f(x)$ and $g(x)$, if $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ exists, then

the value of $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ is

(i) 1

(ii) 0

(iii) $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

(iv) $\lim_{x \rightarrow a+0} \frac{F(x)}{G(x)}$

(Choose the correct option)

(d) Define degree of a differential equation.

(e) Which of the following is not correct ?

(i) $\mu = \frac{1}{2} [E^{1/2} + E^{-1/2}]$

(ii) $\mu^2 = 1 + \frac{1}{4} \delta^2$

(iii) $\mu \delta = \Delta + \nabla$

(iv) $E^{1/2} = \frac{1}{2} \delta + \mu$

(Choose the correct option)

(f) If x is an integer greater than $n-1$,

then $x^{(n)} = \frac{x!}{(x-n)!}$.

(State True or False)

(g) Define Gamma function.

(h) For what purpose, regula falsi method is used ?

(i) Numerical integration

(ii) Determination of the roots of a polynomial

(iii) Solution for differential equation

(iv) None of the above

(Choose the correct option)

2. Answer **any six** questions from the following :
2×6=12

(a) Define General and Particular Solutions of a differential equation.

(b) Solve the difference equation

$$U_{x+1} - aU_x = 0, a \neq 1.$$

(c) Obtain the function whose first difference is $9x^2 + 11x + 5$.

(d) Prove that $E \equiv e^{hD}$.

(Symbols have their usual meanings)

(e) Solve the following differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

(f) Show that the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots$ is not convergent.

(g) State Raabe's test for convergence of a series.

(h) Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$

(i) If $x = r \cos \theta$ and $y = r \sin \theta$, find the Jacobian of the transformations of variables (x, y) to (r, θ) .

(j) Find the value of $\beta(1/2, 1/2)$.

3. Answer **any four** questions from the following :
5×4=20

(a) Express the function $f(x) = 2x^3 - 3x^2 + 3x - 10$ in a factorial notation and hence show that $4^3 f(x) = 12$.

(b) If $x = u - v + w$
 $y = u^2 - v^2 - w^2$
 $z = u^3 + v$

then evaluate the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$

- (c) Prove that $\beta(m, n) = \beta(n, m)$.
- (d) Define positive term series. Show that the positive term geometric series $1 + r + r^2 + \dots$ converges for $r < 1$.

- (e) Show that $\Delta^n 0^m = n(\Delta^{n-1} 0^{m-1} + \Delta^n 0^{m-1})$ where n and m are positive integer.

- (f) Find the maximum and minimum values of the function

$$f(x) = 8x^5 - 15x^4 + 10x^2$$

- (g) Solve the difference equation

$$u_{x+2} - 4u_{x+1} + 4u_x = 2^x$$

- (h) Solve the differential equation

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

4. Answer **any two** questions from the following:
 $10 \times 2 = 20$

- (a) (i) State and prove Bessel's central difference formula. 5

- (ii) If third differences are constant, prove that

$$y_{x+1/2} = \frac{1}{2}(y_x + y_{x+1}) - \frac{1}{16}(\Delta^2 y_{x-1} + \Delta^2 y_x)$$

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- (b) (i) Discuss the convergence of Regula Falsi method.

- (ii) Prove that $\Gamma(1/4)\Gamma(3/4) = \sqrt{2\pi}$.

- (c) (i) State the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact. 1

- (ii) Solve the differential equation $(x^2 + y^2 + x)dx + xy dy = 0$. 5

- (iii) Obtain the differential equation of the family of curves

$$y = e^x (A \cos x + B \sin x). \quad 4$$

- (d) (i) Write a note on Lagrange's undetermined multipliers. 5

- (ii) Show that if $2x + 3y + 4z = a$, the maximum value of $x^2 y^3 z^4$ is $\left(\frac{a}{9}\right)^9$. 5

- (e) (i) Show that

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \beta(m, n)$$

for $m, n > 0$. 5

- (ii) If a function $f(x)$ has maximum or minimum at a point $x = a$ within its domain, then show that $f'(a) = 0$. 5
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