3 (Sem – 1) PHY M1

2020

(Held in 2021)

PHYSICS

(Major)

Paper : 1·1

(Mathematical Method – I and Mechanics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Group – A

(Mathematical Method)

Marks : 20

1. *(a)* What is the geometrical interpretation of the vector product of two vectors ? 1

(b) Prove the vectors $\vec{A} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = \hat{i} + 4\hat{j} - 3\hat{k}$ are perpendicular to each other. 1

(c) If $\vec{r}$ is the position vector of a point, then show that $\vec{\nabla}.\vec{r} = 3$. 1

(d) What is solenoidal vector ? 1

2. (a) What is the physical significance of divergence of a vector ? 2

(b) If $\vec{A}$ and $\vec{B}$ are irrotational, then prove that $\vec{A} \times \vec{B}$ is solenoidal. 2

(c) If the non-parallel vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude, show that vector $\left(\vec{A} + \vec{B}\right)$ is perpendicular to vector $\left(\vec{A} - \vec{B}\right)$. 2

3. Answer ***any two*** of the following : 5×2=10

(a) Prove that

$$\hat{i} \times \left(\vec{A} \times \hat{i}\right) + \hat{j} \times \left(\vec{A} \times \hat{j}\right) + \hat{K} \times \left(\hat{A} \times \hat{K}\right) = 2\vec{A}$$

(b) (i) The electrostatic force acting between two point charges q and q' at a distance r apart is $\vec{E} = \frac{qq'}{r^3}\hat{r}_0$ where $\hat{r}_0$ is unit vector along $\vec{r}$. Find out *curl* of $\vec{E}$.

(ii) Justify the statement that the electric lines of force cannot be closed lines.

(c) An object is attracted to a fixed point O with a central force $\vec{F} = f(r)\vec{r}$, $\vec{r}$ is the position vector of the object relative to point O. Show that $\vec{r} \times \vec{v} = \vec{h}$, where $\vec{h}$ is a constant vector. Prove that the angular momentum is constant.

Group-B

(Mechanics)

Marks : 40

4. (a) A force $\vec{F}$ acts on a body and changes its position from A to B along a path $\vec{r}$ such that $\oint \vec{F}.\overrightarrow{dr} = 0$. What is the nature of the force ? 1

(b) Name the physical quantity which is the cause of rotational motion. Is it a scalar or a vector quantity ? 1

(c) Is the centre-of-mass frame of reference, an inertial frame ? Explain. 1

(d) Name the fictitious force obtained in the rotating frame of reference. 1

5. (a) Distinguish between inertial and non-inertial frames of reference. Is earth an inertial frame ? 2

(b) Show that the force field given by $\vec{F} = x^2yz\hat{i} - xyz^2\hat{k}$ is non-conservative. 2

(c) Give schematic diagram of the two particles collision in laboratory frame and centre of mass frame. 2

6. Answer ***any two*** of the following : 5×2=10

(a) Show that centre of suspension and centre of oscillation of a compound pendulum are interchangeable.

(b) Show that whenever a body is acted upon by a number of forces such that the resultant is not zero, then the work done by the resultant force is equal to the change in the kinetic energy of body.

(c) Prove that conservative force can be expressed as negative gradient of potential.

7. Answer ***any two*** of the following : 10×2=20

(a) *(i)* Show that angular momentum of an extended system is

$$\vec{L} = \vec{L}_{cm} + \vec{R}_{cm} \times \vec{P}$$

where the symbols used in the above expression carry their usual meaning.

(ii) Assuming earth as a sphere, calculate the angular momentum of earth rotating about its own axis of rotation.

Given, mass of earth = $6 \times 10^{24} kg$

radius of earth = $6 \cdot 4 \times 10^{4} m$

6+4=10

(b) Calculate the moment of inertia of solid sphere rotating about its diameter passing through the centre. How can you identify a solid sphere from a hollow sphere of same mass and radius ?

8+2=10

(c) (i) Prove that in centre of mass system the magnitude of the velocities of the particles remain unaltered in elastic collision.

(ii) State the relation between velocities in centre of mass system and laboratory system. 7+3=10

(d) (i) Define coriolis force. Discuss the effect of coriolis force on a particle moving on the surface of the earth. Calculate the vertical and horizontal component of coriolis force.

(ii) Calculate coriolis force on a mass of 50*gm* placed at a distance of 10 *cm* from the axis of a rotating system, if the angular speed of the frame is 10 *rad/sec*. 1+7+2=10

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3 (Sem–1) PHY M2

2020

(Held in 2021)

PHYSICS

(Major)

Paper : 1·2

(Waves, Oscillations and Ray Optics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

SECTION–I

(Waves and Oscillations)

Marks : 40

1. Answer the following questions : 1×4=4

   (a) How is phase of an oscillating particle measured in terms of its time period ? 1

Contd.

(b) How are stationary waves formed ? 1

(c) What is reverberation in sound waves ? 1

(d) What simplification is obtained in the Fourier series if the function is odd ? 1

2. Answer the following : 2×3=6

(a) State the differences between transverse wave and longitudinal wave. 2

(b) What is sharpness of resonance ? Explain the effects of damping on the sharpness of resonance. 2

(c) What is the speed of transverse wave in a rope of length $1m$ and mass $0{\cdot}6kg$ under a tension of $1000N$? 2

3. Answer ***any two*** questions : 5×2=10

(a) Using the method of separation of variables, find the general solution of the differential wave equation in one-dimension. 5

(b) If the relaxation time of a damped harmonic oscillator is 50 second, find the time in which *(i)* the amplitude falls $\frac{1}{e}$ times the initial value *(ii)* energy of the system falls to $\frac{1}{e}$ times the initial value *(iii)* energy falls to $\frac{1}{e^4}$ of the initial value. 5

(c) Find the expression of velocity of a longitudinal wave in a thin solid bar. 5

Answer ***any two*** *questions.*

4. What do you mean by Simple harmonic motion ? Derive and solve the differential equation of a Simple harmonic oscillator. Show that the velocity of the particle is maximum while passing through the mean position. 2+2+4+2=10

5. State Fourier's Theorem. Analyze with the help of Fourier's theorem, a square periodic wave given by

$$y = A(\text{constant}) \text{ for } 0 \le t \le T/2$$
$$= -A(\text{constant}) \text{ for } T/2 \le t \le T$$

Also plot the Fourier synthesis with first four terms. 2+6+2=10

6. Obtain the differential equation for the transverse vibration of a stretched string. Solve the equation by the method of separation of variables. 5+5=10

7. Two simple harmonic motions act simultaneously on a particle at right angles to each other. Show that the path of the particle will be an ellipse when two motions have the same period but different amplitudes and initial phases. What happens when the phase difference between the motion is *(i)* zero and *(ii)* $\frac{\pi}{2}$? 6+2+2=10

SECTION–II

(Ray Optics)

Marks : 20

Answer ***any four*** *questions :* 5×4=20

8. Using Fermat's principle, establish the laws of reflection of light at a plane surface. 5

9. Establish the refraction matrix for the refraction of a ray of light at a spherical surface separating media of refractive indices n_1 and n_2. 5

10. Deduce the Helmholtz equation showing the relation between linear and angular magnification of two conjugate planes in an optical system. 5

11. Show that the combination of two lenses, made of same material, can minimize the chromatic aberration if they are separated by a distance equal to the mean of the focal lengths of two lenses. 5

12. Show that spherical refracting surface is aplanatic with respect to certain position of the object. 5