2017

MATHEMATICS

(Major)

Paper : 2·1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Transform to axes inclined at 45° to the original axes the equation $x^2 - y^2 = a^2$.

(b) Write down the condition for pair of lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

(c) Write down the parametric equations of the parabola.

(d) Write down the direction cosines of x-axis.

(e) About which axis the parabola $y^2 = 4ax$ is symmetric?

(f) Find the eccentricity of the ellipse

$$x^2 + 3y^2 = a^2$$

(g) Write the equation of the diameter of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

parallel to the line $y = mx + c$.

(h) Find the centre and radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y + 2z + 3 = 0$$

(i) Define conjugate planes.

(j) Define enveloping cylinder.

2. Answer the following : 2×5=1

(a) If the axes be turned through an angle $\tan^{-1} 2$, what does the equation $4xy - 3x^2 = a^2$ become?

(b) Find the value of k so that $kxy - 8x + 9y = 12$ may represent pair of straight lines.

(c) Find the equation of the sphere through circles

$$x^2 + y^2 + z^2 = 25, \quad x + 2y - z + 2 = 0$$

and the point (1, 1, 1).

(d) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are the extremities of any focal chord of the parabola $y^2 = 4ax$, prove that $t_1 t_2 = -1$.

(e) The axis of a right circular cylinder is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{2}$$

and its radius is 5. Find the equations.

3. Answer any *two* parts : 5×2=10

(a) By transforming to parallel axes through a properly chosen point (h, k), prove that the equation

$$12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$$

can be reduced to one containing only terms of the second degree.

(b) Reduce the equation

$$7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$$

to the standard form.

(c) Show that the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

represents a pair of parallel straight lines, if

$$\frac{a}{h}=\frac{h}{b}=\frac{g}{f}$$

(d) Prove that the sum of the squares of the reciprocals of two perpendicular diameters of an ellipse is constant.

4. Answer any *two* parts : 5×2=10

(a) Prove that the straight line $y=mx+c$ touches the parabola $y^2=4a(x+a)$, if

$$c=ma+\frac{a}{m}$$

(b) Find the length of the semi-axes of the conic $ax^2+2hxy+ay^2=d$.

(c) Prove that the line $lx+my=n$ is a normal to the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

if $\dfrac{a^2}{l^2}+\dfrac{b^2}{m^2}=\dfrac{(a^2-b^2)^2}{n^2}$.

(d) Find the equation of the tangent to the hyperbola $4x^2 - 9y^2 = 1$ which is parallel to the line $4y = 5x + 7$.

5. Answer any *four* parts : 5×4=20

(a) Find the condition that the homogeneous equation of second degree

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

may represent a pair of planes.

(b) Obtain the shortest distance between the lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

and

$$\frac{x-\alpha'}{l'} = \frac{y-\beta'}{m'} = \frac{z-\gamma'}{n'}$$

(c) Prove that the equation of the plane containing the line

$$\frac{y}{b} + \frac{z}{c} = 1, \quad x = 0$$

and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is

$$\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$$

If $2d$ is the shortest distance between the given lines, prove that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$$

(d) Find the condition when the plane $lx+my+nz=p$ becomes a tangent plane to the sphere

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

(e) Find the equation of the cone whose vertex is at the origin.

(f) Prove that the equation of the polar of the origin with respect to the conic

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

is $gx+fy+c=0$.

6. Answer any *four* parts : 5×4=20

(a) Find the equation of the cylinder whose axis and guiding curve are given.

(b) Find the condition when the plane $lx+my+nz=p$ becomes a tangent plane to the conicoid $ax^2+by^2+cz^2=1$.

(c) Find the equation of the enveloping cone of a conicoid whose vertex is given.

(d) If the section of the enveloping cone of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

whose vertex is P, by the plane $z=0$ is a rectangular hyperbola, prove that the locus of P is

$$\frac{x^2+y^2}{a^2+b^2}+\frac{z^2}{c^2}=1$$

(e) Prove that the locus of the poles of tangent plane of the conicoid $ax^2 + by^2 + cz^2 = 1$ with respect to the conicoid $\alpha x^2 + \beta y^2 + \gamma z^2 = 1$ is the conicoid

$$\frac{\alpha^2 x^2}{a} + \frac{\beta^2 y^2}{b} + \frac{\gamma^2 z^2}{c} = 1$$

(f) Find the equation of the right circular cylinder whose guiding curve is

$$x^2 + y^2 + z^2 = 9$$
$$x - y + z = 3$$

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2017

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : 1×10=10

   (a) Determine the order and degree of the differential equation

   $$K\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

   (b) Define Bernoulli's differential equation.

   (c) If the differential equation $Mdx+Ndy=0$ is homogeneous and $Mx+Ny\neq 0$, write the integrating factor.

   (d) What do you mean by self-orthogonal family of curves?

(e) What is the complementary function of the following differential equation?

$$(D^3 + 3D^2 + 3D + 1)y = e^{-x}$$

(f) Write down the general solution of the differential equation

$$y = px + p - p^2$$

(g) Write down the condition of exactness of a total differential equation

$$Pdx + Qdy + Rdz = 0$$

(h) Find the particular integral of the differential equation

$$(D^2 + a^2)y = \sin ax$$

(i) Write the standard form of the linear partial differential equation of order one.

(j) Find an integral belonging to complementary function of the differential equation

$$y_2 - \cot x y_1 - (1 - \cot x)y = e^x \sin x$$

2. Answer the following questions : $2 \times 5 = 10$

(a) Form the differential equation of which $xy = ae^x + be^{-x}$ (a, b parameters) is a solution.

(b) Solve :

$$(x+y)^2 \frac{dy}{dx} = a^2$$

(c) Solve :

$$\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$$

(d) If $\frac{dy}{dx} + 2y \tan x = \sin x$ and if $y = 0$ when $x = \frac{\pi}{2}$ express y in terms of x.

(e) Construct the partial differential equation by eliminating a and b from

$$z = (x^2 + a)(y^2 + b)$$

3. Answer any *four* questions : 5×4=20

(a) Prove that a necessary and sufficient condition that the differential equation $Mdx + Ndy = 0$ be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(b) Find the orthogonal trajectories of the family of curves $x^2 + y^2 = 2ax$, a being parameter.

(c) Solve :

$$\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$$

(d) Apply variation of parameter to solve the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$$

(e) Solve :

$$x^3\frac{d^3y}{dx^3}+6x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}+2y=x^2+3x-4$$

(f) Obtain the general and singular solution of the differential equation

$$y=px+\sqrt{b^2+a^2p^2}$$

4. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Solve :

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x\cos x$$

(b) Solve :

$$t\,dx=(t-2x)\,dt$$
$$t\,dy=(tx+ty+2x-t)\,dt$$

(c) Solve :

$$\frac{dy}{dx}=x^3y^3-xy$$

(d) Reduce the equation $y^2(y-px)=x^4p^2$, where $p=\dfrac{dy}{dx}$ to Clairaut's form by the substitution $x=\dfrac{1}{X}$, $y=\dfrac{1}{Y}$ and hence solve the equation.

5. Answer *either* (a) and (b) *or* (c) and (d) : 5+5=10

(a) Find the necessary condition for integrability of the total differential equation $Pdx+Qdy+Rdz=0$.

(b) Reduce the differential equation

$$\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$$

to the form $\dfrac{d^2v}{dx^2}+Q_1v=R_1$, where Q_1 and R_1 are functions of x to solve the differential equation.

(c) Solve :

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2$$

by changing the independent variable x to z.

(d) Solve :

$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x(1-x^2)^{3/2}$$

6. Answer *either* *(a)* and *(b)* or *(c)* and *(d)* : 5+5=1

(a) Solve :

$$\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$$

(b) Solve :

$$3x^2dx+3y^2dy-(x^3+y^3+e^{2z})dz=0$$

(c) Show that the differential equation

$$xdx+ydy=\frac{a^2(x\,dy-y\,dx)}{x^2+y^2}$$

is exact and hence solve it.

(d) Find $f(z)$ such that

$$\left(\frac{y^2+z^2-x^2}{2x}\right)dx-ydy+f(z)dz=0$$

is integrable and hence solve it.

7. Answer *either* *(a)* and *(b)* or *(c)* and *(d)* : 5+5=10

(a) Solve by Lagrange's method

$$z(x+y)p+z(x-y)q=x^2+y^2$$

(b) Solve by Charpit's method

$$pxy+pq+qy-yz=0$$

(c) Find the complete integral of $9(p^2z+q^2)=4$. Also find the singular solution if it exists.

(d) Derive the partial differential equation by the elimination of arbitrary function from the equation $\phi(u, v)=0$, where u and v are functions of x, y and z.

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