2012

MATHEMATICS

(Major)

Paper : 2.1

(Coordinate Geometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) What is the locus represented by the equation $x^2 - 5xy + 6y^2 = 0$? 1

(b) Write down the formulae of transformation from one pair of rectangular axes to another with same origin. 1

(c) What will be the equation of the line $ax + by + c = 0$ if the origin is transferred to the point (α, β)? 1

(d) Find the equation to the locus of the point $P(t, 2t)$ if t is a parameter. 1

(e) The parabola represented by the equation $y^2 = 4ax$ is not a closed curve. How can you justify it from the given equation? 1

(f) Write the relationship between the lengths of semi-major axis, semi-minor axis and the eccentricity for the standard equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \; b > a$$ 1

(g) What are the direction cosines of the normal to the plane given by the equation

$$ax + by + cz + d = 0 \, ?$$ 1

(h) Mention the condition under which the lines

$$\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$$

and $$\frac{x - \alpha'}{l'} = \frac{y - \beta'}{m'} = \frac{z - \gamma'}{n'}$$

are coplanar. 1

(i) When is a plane said to be parallel to a line? 1

(j) What are centre and radius of the sphere given by the equation

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \text{ ?}$$ 1

2. (a) Find the transformed equation of the line $y = x$ when the axes are rotated through an angle 45°. 2

(b) Mention the conditions under which the general equation of the second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents (i) a parabola, (ii) an ellipse, (iii) a hyperbola and (iv) a circle. 2

(c) Find the equation of the cone whose vertex is at the origin and whose guiding curve is given by

$$x = a,\ y^2 + z^2 = b^2$$ 2

(d) Find the equation of the sphere through the circle

$$x^2 + y^2 + z^2 = 9,\ 2x + 3y + 4z = 5$$

and the point (1, 2, 3). 2

(e) Find the perpendicular distance of the point (1, 4, – 2) from the plane

$$2x - 3y + z = 5$$ 2

3. (a) Prove that the line $lx + my = n$ is a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{if } \frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$$ 5

(b) Find the polar equation of a conic with a focus as the pole and the line joining the focus to the corresponding vertex as the initial line. Hence deduce the equation if the line joining the focus with the corresponding vertex makes an angle α with the initial line. 4+1=5

Or

Prove that the middle points of chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ parallel to the diameter $y = mx$ lie on the diameter $a^2 my = b^2 x$. 5

(c) Find the centre and radius of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$$
$$x - 2y + 2z = 3$$ 5

Or

The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes in A, B, C. Prove that the equation of the cone generated by the lines drawn from O to meet the circle ABC is

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

(d) Show that the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$$

and guiding curve is $x^2 + 2y^2 = 1$, $z = 3$ is

$$3(x^2 + 2y^2 + z^2) + 8yz - 2zx + 6x - 24y - 18z + 24 = 0$$

5

Or

Prove that from any point six normals can be drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$.

4. (*a*) Find the condition under which the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel straight lines. Find the distance between such two parallel lines. Verify the condition you have derived in case of the equation

$$4x^2 + 12xy + 9y^2 + 8x + 12y = 0$$

and find the equation of the parallel lines. 4+3+1+2=10

Or

Find the condition under which the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of intersecting straight lines. Find the angle between those two intersecting lines and hence find the conditions under which they are parallel and perpendicular. 5+3+1+1=10

(*b*) Show that the equation of the general conic of the form

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \ (ab \neq h^2)$$

can be reduced to the central conic of the form

$$a'x^2 + b'y^2 + d = 0$$

Hence find where to the origin must be shifted so that the equation

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

reduces to a central conic. 7+3=10

Or

Find the equation of a polar of a given point $P(x_1, y_1)$ with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Hence show that if the polar of a point P with respect to the conic passes through a point Q, then the polar of Q also passes through P. 7+3=10

(c) What do you mean by skew lines? How do you define the shortest distance between two such lines? Find the length and the equations of the line of shortest distance between the lines

$$3x - 9y + 5z = 0, \; x + y - z = 0$$

$$6x + 8y + 3z - 13 = 0, \; x + 2y + z - 3 = 0$$

2+2+6=10

Or

Find where the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

meets the plane

$$2x + 4y - z + 1 = 0$$

and find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$$ 5+5=10

(d) What do you mean by an enveloping cone? Find the equation of the enveloping cone with respect to the conicoid $ax^2 + by^2 + cz^2 = 1$ with (α, β, γ) as the vertex of the enveloping cone. 2+8=10

Or

What do you mean by a director sphere? Find the equation of the director sphere of the conicoid $ax^2 + by^2 + cz^2 = 1$. Hence or otherwise prove that the director sphere of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is $x^2 + y^2 + z^2 = a^2 + b^2 + c^2$. 2+7+1=10

★★★

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3 (Sem–2) MAT M 2

2012

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following : $1\times10=10$

(a) Determine the order, degree, unknown functions and independent variable of the differential equation

$$5\left(\frac{d^4b}{dp^4}\right)^5+7\left(\frac{db}{dp}\right)^{10}+b^7-b^5=p$$

(b) Which of the following functions is solution of the differential equation $y_2-y=0$?

(i) $y=e^x$

(ii) $y=\sin x$

(iii) $y=4e^{-x}$

(iv) $y=\tan x$

(c) When the first-order and first-degree differential equation

$$M\,dx + N\,dy = 0$$

(M and N are functions of x and y) is said to be exact?

(d) Write down the general solution of the differential equation

$$y = px + \sqrt{a^2 p^2 + b^2}, \quad p = \frac{dy}{dx}$$

(e) Find an integral belonging to complementary function of the differential equation

$$x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$$

(f) Write down the necessary condition for integrability of single differential equation

$$Pdx + Qdy + Rdz = 0$$

(g) Choose the key giving the correct answer :

The partial differential equations can be formed by the elimination of

(i) arbitrary constants only

(ii) arbitrary functions only

(iii) arbitrary functions or arbitrary constants

(iv) None of the above

(h) Write down the particular integral of the differential equation

$$y_2 + 3y_1 + 2y = e^x$$

(i) Give the geometrical interpretation of the differential equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

(P, Q and R are functions of x, y and z)

(j) Construct the partial differential equation by eliminating a and b from

$$z = ax + (1-a)y + b$$

2. Answer the following : 2×5=10

(a) Determine c_1 and c_2 so that

$$y(x) = c_1 e^{2x} + c_2 e^x + 2\sin x$$

will satisfy the condition $y(0) = 0$ and $y'(0) = 1$.

(b) Solve :

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

(c) Solve :

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

(d) Solve :

$$\frac{dx}{dt} = x^2 - 2x + 2$$

(e) Find the differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$ for different values of A and B.

3. Answer any *four* parts : 5×4=20

(a) Find the orthogonal trajectories of the family of curves

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + k} = 1$$

k being the parameter.

(b) Solve by the method of variation of parameter :

$$y_2 - y = \frac{2}{1 + e^x}$$

(c) Solve

$$\left(\frac{dy}{dx}\right)(x^2 y^3 + xy) = 1$$

given $y = 0$ when $x = 1$.

(d) Find a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x + y + z, x^2 + y^2 - z^2) = 0$$

(e) Solve the differential equation

$$\sin^2 x\left(\frac{d^2y}{dx^2}\right) = 2y$$

given that $y = \cot x$ is a solution.

(f) Show that the differential equation of all cones which have their vertices at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

4. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve :

$$(D^2 + 3D + 2)y = e^{2x} \sin x$$

(b) Solve :

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

(c) Solve :

$$p^2 + 2py \cot x = y^2$$

(d) Show that the following equation is exact and hence solve it :

$$\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\} dx + \{x + \log x - x \sin y\}\, dy = 0$$

5. Answer *either* (a) *or* (b) and (c) : 5+5

(a) Reduce the differential equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$$

to the form $\dfrac{d^2v}{dx^2} + Iv = S$

(I, S are functions of x) to solve the differential equation. Hence solve the following equation :

$$y_2 - 4xy_1 + (4x^2 - 3)y = e^{x^2}$$

(b) If $y = y_1(x)$ and $y = y_2(x)$ are two solutions of the equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

prove that

$$y_1 \frac{dy_2}{dx} - y_2 \frac{dy_1}{dx} = ce^{-\int P dx}$$

c being an arbitrary constant.

(c) Solve the differential equation

$$x^6 y'' + 3x^5 y' + a^2 y = \frac{1}{x}$$

by changing the independent variable x to z.

6. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Find $f(y)$ such that the total differential equation

$$\left(\frac{yz + z}{x}\right) dx - z dy + f(y) dz = 0$$

is integrable. Hence solve it.

(b) Solve :

$$xz^3 dx - z dy + 2y dz = 0$$

(c) Solve :

$$\frac{dx}{dt} + 5x + y = e^t$$
$$\frac{dy}{dt} - x + 3y = e^{2t}$$

(d) Solve the simultaneous equations :

$$\frac{adx}{yz(b-c)} = \frac{bdy}{zx(c-a)} = \frac{cdz}{xy(a-b)}$$

7. Answer *either* (a) and (b) *or* (c) and (d) : 5+5

(a) Solve by Lagrange's method :

$$p + q = x + y + z$$

(b) Find the integral surface of the partial differential equation

$$(x-y)p + (y-x-z)q = z$$

through the circle $z = 1,\ x^2 + y^2 = 1$.

(c) Solve by Charpit's method :

$$(p^2 + q^2)y = qz$$

(d) Find the complete integral of

$$z^2(p^2z^2 + q^2) = 1$$

Find also the singular integral if it exists.

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