2017

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods–II)

(*Marks* : 35)

1. Answer the following questions : 1×3=3

   (a) Define surface integral of a vector.

   (b) Write down the unit vectors $\hat{e}_\rho$, $\hat{e}_\phi$ and $\hat{e}_z$ of a cylindrical coordinate system in terms of $\hat{i}$, $\hat{j}$, $\hat{k}$.

   (c) Give graphical representation of Dirac delta function.

2. If $\vec{V} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, then evaluate $\int_C \vec{V} \cdot d\vec{r}$, where C is a straight line joining (0, 0, 0) and (1, 1, 1). 2

3. Answer any *two* of the following questions : 5×2=10

(a) *(i)* Show that $\Gamma(n+1) = n\Gamma(n)$. 2

(ii) Find the value of $\Gamma(\frac{1}{2})$. 3

(b) *(i)* Show that $\iint_S \vec{r} \cdot \hat{n}\, dS = 3$, over the surface S of the unit cube bounded by coordinate planes and the planes $x = 1$, $y = 1$, $z = 1$. 3

(ii) From the definition of Dirac delta function, show that

$$\int_{-\infty}^{+\infty} f(x)\, 8(x-a)\, dx = f(a)$$ 2

(c) *(i)* Express differential operator $\vec{\nabla}$ in terms of orthogonal curvilinear coordinates. 3

(ii) If $\vec{r} = (u, v)$ represents a surface, then show that the square of the element of arc length on the

surface is

$$dS^2 = E\,du^2 + 2F\,du\,dv + G\,dv^2$$

where $E = \frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial \vec{r}}{\partial u}$, $F = \frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial r}{\partial v}$ and

$G = \frac{\partial \vec{r}}{\partial v} \cdot \frac{\partial \vec{r}}{\partial v}$. 2

4. Answer any *two* of the following questions : 10×2=20

(a) (i) If V be the volume bounded by a closed surface S and $\vec{A}$ is a vector function of position with continuous derivatives, then prove that

$$\oiint_S \vec{A} \cdot \hat{n}\, dS = \iiint_V (\vec{\nabla} \cdot \vec{A})\, dV$$ 5

(ii) Prove the following identity : 3

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dV = \iint_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) \cdot d\vec{S}$$

(iii) If $\hat{n}$ be the unit outward drawn normal to any closed surface S, find the volume integral

$$\iiint_V \vec{\nabla} \cdot \hat{n}\, dV$$ 2

(b) (i) Express elements of area in orthogonal curvilinear coordinates. 3

(ii) Represent the vector

$$\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$$

in spherical polar coordinate. 5

(iii) Find the volume of a sphere of radius R. 2

(c) (i) Using vector integration, prove the equation of

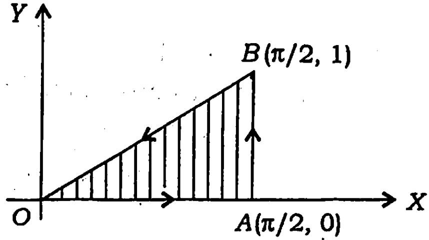
$$\vec{\nabla} \cdot (\rho\vec{v}) + \frac{\partial \rho}{\partial t} = 0$$

where ρ and $\vec{v}$ are the density and velocity of the fluid respectively. 5

(ii) State Green's theorem in the plane and using it evaluate

$$\oint_C (y - \sin x)\, dx + \cos x\, dy$$

where C is the triangle of the following figure : 5



GROUP—B

(Properties of Matter)

(*Marks* : 25)

5. Answer the following questions : 1×4=4

(a) Write down the relation connecting elastic constants Y, K and η.

(b) A rod is suspended vertically with its upper end rigidly fixed and is twisted at its lower end by means of a couple. For which layer of cylindrical rod shear is maximum?

(c) Why is Poiseuille's equation is not valid for blood flows through veins and capillaries?

(d) Define neutral surface of a beam.

6. Answer the following questions : 2×3=6

(a) Show that for a homogeneous and isotropic material the value of Poisson's ratio lies between −1 and ½, and is equal to ½ only when the material is incompressible.

(b) Write down the Poiseuille's assumptions to determine the rate of flow of liquid through a tube.

(c) How much work is required to break up a liquid drop of radius R into n equal small drops?

7. Answer any *one* of the following questions : 5

(a) *(i)* Show that simultaneous equal compression and extension at right angles to each other are equivalent to a shear. 3

(ii) Find the work done in joules in stretching a wire of cross-section 1 sq. mm and length 2 meters through 0·1 mm, if Young's modulus for the material of the wire is 2×10^{12} dynes/cm^2. 2

(b) *(i)* Find an expression for bending moment of a beam. 3

(ii) A steel rod of length 50 cm, width 2 cm and thickness 1 cm is bent into the form of an arc of radius of curvature 2 m. Calculate the bending moment. Young's modulus of the rod = 2×10^{11} N/m^2. 2

8. Answer *either (a)* and *(b) or (c)* and *(d)* : 10

Either

(a) Establish the relation between surface energy E and surface tension S of a liquid $E = S - T\frac{dS}{dT}$. 5

(b) *(i)* Show that angular oscillation of a torsional pendulum is simple harmonic and hence find the time period of the pendulum. 2

(ii) How can moment of inertia of a body be determined with the help of torsional pendulum? 3

Or

(c) Describe with necessary theory the rotating cylinder experiment for the measurement of coefficient of viscosity. 7

(d) An aluminium wire of length 2 m and radius 1 mm is twisted through 90°. Find the angle of shear at the surface, at the axis of the wire and at a point midway between the axis and surface. Calculate the torsional couple, if the modulus of rigidity is 5×10^{10} N/m^2. 3

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2017

PHYSICS

(Major)

Paper : 2.2

(Heat and Thermodynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Symbols have their usual meanings

1. Answer the following questions : 1×7=7

(a) What is the magnitude of γ (the ratio of two specific heat) for a diatomic molecule?

(b) State the law of corresponding state of van der Waals' gas.

(c) Write down the relation between pressure and energy density of diffuse radiation.

(d) Under what condition will the efficiency of Carnot engine be 100%? Is it possible?

(e) Show that the energy of Planck's oscillator

$$\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$$

reduces to equipartition law of energy kT at high temperature.

(f) What is the magnitude of specific heat of water in SI system?

(g) A volume of a gas expands isothermally to four times its initial volume. Calculate the change of entropy in terms of gas constant.

2. Answer any *four* questions : $2\times4=8$

(a) Starting from the expression of pressure exerted by perfect gas, deduce Clapeyron's equation $p = nkT$.

(b) Using Maxwell's velocity distribution law, deduce an expression for most probable velocity of gas molecules of a perfect gas.

(c) Calculate the amount of work done during adiabatic expansion of a gas.

(d) If an spherical enclosure full of radiation is allowed to expand adiabatically, show that the radiation behave like a gas having $\gamma = \frac{4}{3}$.

(e) Using van der Waals' equation, show that at critical point, the volume of a gas V_c is equal to three times of van der Waals' constant b, i.e., $V_c = 3b$.

3. Answer any *three* questions : 5×3=15

(a) State and deduce the Kirchhoff's law regarding blackbody radiation.

(b) Show that the Joule-Thomson coefficient μ for an ideal gas is zero and for van der Waals' gas

$$\mu = \frac{1}{C_p}\left[\frac{2a}{RT} - b\right]$$

(c) Establish the relation $TV^{\gamma-1}$ = const., for adiabatic expansion of a perfect gas.

(d) The mean KE of a molecule of hydrogen gas at 0 °C is $5 \cdot 62 \times 10^{-21}$ J and molar gas constant $R = 8 \cdot 31$ JK^{-1}. Calculate Avogadro's number and Boltzmann constant k.

(e) Calculate the increase in entropy, when 1 gram ice at – 10 °C is converted into steam at 100 °C. Given specific heat of ice is 0·5 cal/gram/°C, latent heat of ice is 80 cal/gram and latent heat of steam is 540 cal/gram.

4. Answer any *three* of the following : 10×3=30

(a) State Stefan's law of blackbody radiation. Obtain this law from Planck's law of radiation . Given that

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$$

(b) Establish the following relation :

(i) $C_p - C_v = R\left(1 + \frac{2a}{VRT}\right)$ for van der Waals' gas

(ii) $C_p - C_v = -TE\alpha^2 V$

(c) Using kinetic theory of gases, show that the number of molecules in the energy range E and $E + dE$ is given by

$$dN_E = 2N\left(\frac{E}{\pi}\right)^{\frac{1}{2}} (kT)^{\frac{3}{2}} e^{-E/kT} dE$$

(d) Discuss the Einstein's theory of translational Brownian motion and derive an expression for average displacement of a particle under Brownian motion.

(e) Deduce Fourier equation for heat conduction in a rectangular bar, when radiation loss is taken into account and hence find a solution of this equation. What is thermometric conductivity?

(f) Define Kelvin absolute scale of temperature. Show that this scale agree with that of perfect gas scale. Negative temperature is not possible on this scale. Discuss.

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