2026

MATHEMATICS

(Major)

Paper : MAT0600104

(Linear Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×8=8

(a) What is the rank of

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}?$$

(b) The set consisting of only the zero vector in a vector space V is a ____ of V.

(Fill in the blank)

(c) Find $\|(0, 5)\|$.

(d) Is the set $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$ a subspace of $\mathbb{R}^2$?

(e) Does the set $\{(1, 0), (0, 1), (1, 1)\}$ form a basis for $\mathbb{R}^2$?

(f) If a 3×3 matrix has a rank of 2, what is the dimension of its null space?

(g) Is the map $T : V_3 \to V_3$ defined by $T(x, y, z) = (x, y)$ linear?

(h) Every constant map from one vector space to another is both one-one and onto.

(State True or False)

2. Answer any *six* questions : 2×6=12

(a) Test whether (1, 2, 3) belongs to the null space of

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

(b) Find the orthogonal projection of (2, 0) on (1, 0).

(c) Write the characteristic polynomial of

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

(d) Explain the relationship between basis and dimension.

(e) Compute the inner product of $u = (1, 2, 3)$ and $v = (2, 0, -1)$.

(f) Find the dimension of span $\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$.

(g) Find the orthogonal projection of $y = (2, 3)$ onto the subspace spanned by $u = (1, 1,)$.

(h) In $\mathbb{R}^3$, find an orthogonal basis for the subspace spanned by (1, 1, 0), (1, 0, 1).

(i) Prove that the intersection of two subspaces of a vector space is a subspace. Give an example in $\mathbb{R}^2$ to show that the union of two subspaces is not in general a subspace.

(j) Let

$$A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$$

An eigenvalue of A is 2. Find a basis for the corresponding eigenspace.

3. Answer any *four* questions : 5×4=20

(a) Find a basis for the null space of

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \end{bmatrix}$$

(b) Show that the set {(1, 0, 1), (2, 1, 3), (0, 1, 1)} is linearly independent.

(c) Find the eigenvalues and eigenvectors of

$$B = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$$

(d) Determine whether the matrix

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

is diagonalizable. Justify.

(e) Apply the Gram-Schmidt process to (1, 1, 0) and (1, 0, 1).

(f) Verify whether $< u, v > = u_1 v_1 + u_2 v_2$ defines an inner product on $\mathbb{R}^2$.

(g) If $v_1, v_2, \cdots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_r$ of an $n \times n$ matrix A, then prove that the set $\{v_1, v_2, \cdots, v_r\}$ is linearly independent.

(h) Diagonalize the following matrix :

$$\begin{bmatrix} -6 & 4 & 0 & 9 \\ -3 & 0 & 1 & 6 \\ -1 & -2 & 1 & 0 \\ -4 & 4 & 0 & 7 \end{bmatrix}$$

4. Answer any *two* questions : 10×2=20

(a) State Cayley-Hamilton theorem. Verify the theorem for the matrix

$$C = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

Use the theorem to find C^{-1}. 2+4+4=10

(b) Define rank and nullity of a matrix. State and prove Rank theorem.

1+1+2+6=10

(c) Define inner product space. Show that the following is an inner product in $\mathbb{R}^2$:

$$< u, v > = 4u_1 v_1 + 5u_2 v_2$$

where

$$u = (u_1, u_2), v = (v_1, v_2) \in \mathbb{R}^2$$

Also, show that in any inner product space V

$$|< u, v >| \leq ||u||.||v||, \forall u, v \in V$$ 2+4+4=10

(d) Given

$$A = \begin{bmatrix} 2 & -1 & 3 & 1 & -2 \\ 1 & 0 & 2 & -1 & 1 \\ 3 & -1 & 5 & 0 & -1 \end{bmatrix}$$

(i) Determine a spanning set for the null space of A.

(ii) Find the dimension of the null space.

(iii) Is the spanning set linearly independent? Justify your answer.

4+3+3=10

(e) The vectors $v_1 = (1, 2, 2)$, $v_2 = (2, 0, 1)$ are given.

(i) Apply the Gram-Schmidt process to the vectors v_1 and v_2.

(ii) Find an orthogonal basis for the subspace spanned by v_1 and v_2.

(iii) Normalize the vectors to obtain an orthonormal basis. 4+3+3=10

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2026

MATHEMATICS

(Major)

Paper : MAT0600204

(Partial Differential Equations)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. (a) Under what condition the partial differential equation

$$a(x, y)u_x + b(x, y)u_y + c(x, y)u = d(x, y)$$

is said to be homogeneous? 1

(b) Classify the partial differential equation

$$u_{xx} + u_{xy} + u_{yy} + u_x = 0$$ 1

(c) Find the complete integral of

$$z = px + qy + f(p, q)$$ 1

(d) If u is the complementary function and z_1 is a particular integral of a linear partial differential equation, then what will be the general solution of this equation? 1

(e) What will be the Charpit's auxiliary equations for a partial differential equation $f(x, y, z, p, q) = 0$? 1

2. Answer any *five* parts : 2×5=10

(a) Construct a partial differential equation by eliminating a and b from

$$z = ax + by + a^2 + b^2$$

(b) Define a partial differential equation and its order.

(c) What is meant by linear and semi-linear partial differential equation?

(d) Show that the family of spheres $x^2 + y^2 + (z-c)^2 = r^2$ satisfies the first-order linear partial differential equation.

(e) Find the general solution of the first-order linear partial differential equation $xu_x + yu_y = u$.

(f) Find a complete integral of $q = 3p^2$.

(g) Find a complete integral of $(p+q)(z - xp - yq) = 1$.

(h) Reduce the equation $u_x - u_y = u$ to canonical form.

(i) Solve $a(p+q) = z$.

(j) Find a complete integral of $z = pq$.

3. Answer any *four* parts : 5×4=20

(a) Solve $z(xp - yq) = y^2 - x^2$.

(b) Eliminate the arbitrary function f from $f(x^2 + y^2 + z^2,\ z^2 - 2xy) = 0$ to form a partial differential equation.

(c) Find the general integrals of the linear partial differential equation

$$y^2 p - xyq = x(z - 2y)$$

(d) Find the general integral of $xzp + yzq = xy$.

(e) Reduce the equation $yu_x + u_y = x$ to canonical form and obtain the general solution.

(f) Solve the initial value problem $u_x + 2u_y = 0$, $u(0, y) = 4e^{-2y}$ by separation of variables.

(g) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

(h) Solve the equation $r + s - 2t = e^{x+y}$ where $r = \frac{\partial^2 z}{\partial x^2}$, $s = \frac{\partial^2 z}{\partial x \partial y}$, $t = \frac{\partial^2 z}{\partial y^2}$.

4. Answer any *one* part : 10

(a) Find complete integrals of the equations using Charpit's method : 5+5=10

(i) $p^2x+q^2y=z$

(ii) $pq=px+qy$

where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$

(b) Determine the general solution of $4u_{xx}+5u_{xy}+u_{yy}+u_x+u_y=2$.

(c) Find the integral surface of the linear partial differential equation

$$x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$$

which contains the straight line $x+y=0$, $z=1$.

(d) Find the complete integral of

$$p_1^3+p_2^2+p_3=1$$

using Jacobi's method.

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MATHEMATICS

(Major)

Paper : MAT0600304

(Metric Space)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×8=8

(a) Describe an open ball in the discrete metric space (X, d).

(b) Define Euclidean metric on $\mathbb{R}^n$.

(c) Find the derived set of

$$F = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right\}$$

(d) Let F_1 and F_2 be two subsets of a metric space (X, d). Then

(i) $\overline{F_1 \cap F_2} = \overline{F_1} \cap \overline{F_2}$

(ii) $(F_1 \cap F_2)' = F_1' \cap F_2'$

(iii) $(A \cup B)^\circ = A^\circ \cup B^\circ$

(iv) $(F_1 \cup F_2)' = F_1' \cup F_2'$

(Choose the correct option)

(e) "Cauchy sequences are mapped into Cauchy sequences by a continuous function defined from a metric space to another metric space."

(Write True or False)

(f) The intersection of an infinite number of open sets need not be open. Justify your answer.

(g) Which of the following statements is not true?

(i) Discrete metric space is a complete metric space.

(ii) In a metric space, any intersection of closed sets is closed.

(iii) A Cauchy sequence in a metric space is convergent in it.

(iv) In the discrete metric space, each subset is open.

(Choose the correct option)

(h) Consider the following statements :

I. In a connected metric space (X, d), there exists a proper subset of X that is both open and closed in X.

II. The continuous image of a connected metric space is connected.

In the light of the above statements

(i) both I and II are true

(ii) both I and II are false

(iii) I is true but II is false

(iv) I is false but II is true

(Choose the correct option)

2. Answer any *six* of the following questions : 2×6=12

(a) Prove that a convergent sequence in a metric space is a Cauchy sequence.

(b) Show that each subset of a discrete metric space has no limit point.

(c) Show that the interior of the subset $[0, 1] \subseteq \mathbb{R}$ is the open interval $(0, 1)$.

(d) Let F be a subset of a metric space (X, d). Then prove that F is closed if and only if $F = \bar{F}$.

(e) If F_1 and F_2 are two subsets of a metric space (X, d), then prove that

$$\overline{F_1 \cup F_2} = \overline{F_1} \cup \overline{F_2}$$

(f) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f : X \to Y$. If $f(\overline{A}) \subseteq \overline{f(A)}$ for all subsets A of X, then prove that f is continuous on X.

(g) Show that the function $f : [0, 1] \to \mathbb{R}$ defined by $f(x) = x^2$ is uniformly continuous on the metric space $[0, 1]$ with the absolute value metric.

(h) Define homomorphism of metric space and give one example.

(i) Show that the metric spaces (X, d_1) and (X, d_2), where

$$d_2(x, y) = \frac{d_1(x, y)}{1 + d_1(x, y)}, \quad \forall\ x, y \in X$$

are equivalent.

(j) Show that the set of rationals $Q \subseteq \mathbb{R}$ with the usual metric induced from $\mathbb{R}$ is not connected.

3. Answer any *four* of the following questions : 5×4=20

(a) Let

$$X = \mathbb{R}^n = \{x = (x_1, x_2, \ldots, x_n) : x_i \in \mathbb{R},\ 1 \le i \le n\}$$

be the set of real n-tuples. For $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ in $\mathbb{R}^n$, define

$$d(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2 \right)^{1/2}$$

Verify that d is a metric on $\mathbb{R}^n$.

(b) Define an open ball in a metric space. Prove that in any metric space each open ball is an open set. 1+4=5

(c) Let (X, d) be a metric space and $Y \subseteq X$. Prove that if X is separable, then Y with the induced metric is separable too.

(d) Let (X, d_X) and (Y, d_Y) be two metric spaces. Prove that a mapping $f : X \to Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y.

(e) Define uniformly continuous mapping on a metric space. Show that the function $f:(0, 1) \to \mathbb{R}$ defined by $f(x) = \frac{1}{x}$ is not uniformly continuous. 1+4=5

(f) Let (X, d) be a metric space and let $x \in X$ and $A \subseteq X$ be non-empty. Prove that $x \in \bar{A}$ if and only if $d(x, A) = 0$.

(g) Let $T: X \to X$ be a contraction of the complete metric space (X, d). Then prove that T has a unique fixed point.

(h) Let (X, d) be a metric space and let $\{Y_\lambda : \lambda \in \Lambda\}$ be a family of connected sets in (X, d) having a non-empty intersection. Prove that $Y = \bigcup_{\lambda \in \Lambda} Y_\lambda$ is connected.

4. Answer any *two* of the following questions : 10×2=20

(a) *(i)* Prove that the metric space $X = \mathbb{R}^n$ with the metric given by

$$d_p(x, y) = \left(\sum_{i=1}^{n} |x_i - y_i|^p \right)^{\frac{1}{p}}, \quad p \geq 1$$

where $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$ are in $\mathbb{R}^n$, is a complete metric space. 6

(ii) Let F be a subset of the metric space (X, d). Prove that F' is a closed subset of (X, d). 4

(b) *(i)* Let (X, d) be a metric space and $F \subseteq X$. Then prove that F is closed in X if and only if F^c is open in X. 5

(ii) Let (X, d) be a metric space and Y be a subspace of X. Let $Z \subseteq Y$. Prove that Z is open in Y if and only if there exists an open set $G \subseteq X$ such that $Z = G \cap Y$. 5

(c) *(i)* If A is a subset of the metric space (X, d), then prove that $d(A) = d(\bar{A})$. 4

(ii) Let (X, d_X) and (Y, d_Y) be metric spaces and $A \subseteq X$. Prove that a function $f: A \to Y$ is continuous at $a \in A$ if and only if, whenever a sequence $\{x_n\}$ in A converges to a, the sequence $\{f(x_n)\}$ converges to $f(a)$. 6

(d) *(i)* Let (X, d) be a metric space and $F \subseteq X$. Then prove that x_0 is a limit point of F if and only if it is possible to select from the set F a sequence of distinct points $x_1, x_2, ..., x_n$ such that

$$\lim_{n} d(x_n, x_0) = 0$$ 5

(ii) Prove that a mapping $f: X \to Y$ is continuous on X if and only if $f^{-1}(F)$ is closed in X for all closed subsets F of Y where (X, d_X) and (Y, d_Y) are two metric spaces. 5

(e) *(i)* Define discrete two-point space. Let (X, d) be a metric space. Prove that the following statements are equivalent : 1+5=6

1. (X, d) is disconnected.
2. There exists a continuous mapping of (X, d) onto the discrete two-element space (X_0, d_0).

(ii) If Y is a connected set in a metric space (X, d), then prove that any set Z such that $Y \subseteq Z \subseteq \overline{Y}$ is connected. 4

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MATHEMATICS

(Major)

Paper : MAT0600404

(Mechanics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) Two equal forces act on a particle. Find the angle between the forces when the square of their resultant is equal to three times of their product.

(b) What is the physical significance of moment of a force about a point?

(c) Give one example of a couple from daily life.

(d) Define amplitude of a simple harmonic motion.

(e) What is the centre of gravity of a uniform triangular lamina?

(f) Define limiting equilibrium and limiting friction.

(g) State Newton's law of motion from which we derive the method of measuring force.

(h) Define impulsive force with an example.

2. Answer the following questions (any *six*) : 2×6=12

(a) P and Q are like-parallel forces. If P is moved parallel to itself through a distance x, then show that the resultant of P and Q moves through a distance $\frac{Px}{P+Q}$.

(b) Prove that the algebraic sum of the moments of the forces forming a couple about any point in their plane is a non-zero constant and equal to the moment of the couple.

(c) If the radial and transverse velocities of a particle are always proportional to each other, then show that the path is an equiangular spiral.

(d) Find the position of centre of gravity of a uniform semicircular arc of radius a.

(e) A point in a straight line with SHM has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is

$$2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(f) State the laws of static friction.

(g) Write down the general conditions of equilibrium of a system of coplanar forces acting on a rigid body.

(h) A particle falls under gravity, supposed constant, in a resisting medium whose resistance varies as the square of the velocity. Find the terminal velocity of the motion.

(i) Prove that the change of kinetic energy of a body is equal to the work done by the acting force.

(j) Find the magnitude of the force which, acting on a mass of 18 kg for 10 seconds, will generate in it a velocity of 5 km per hour.

3. Answer the following questions (any *four*) : 5×4=20

(a) Forces of magnitudes P, $2P$, $-P$, $2P$ act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ of the square $ABCD$, and a force of magnitude $P\sqrt{2}$ acts along each of $\overrightarrow{BD}$ and $\overrightarrow{CA}$. Show that the forces reduce to a couple of moment $2aP$, where a is the side of the square.

(b) Establish the following statement :

Any system of coplanar forces acting on a rigid body can ultimately be reduced either to a single force or to a single couple unless it is in equilibrium.

(c) A particle moves along a straight line, and at a distance x from a fixed point O on the line its velocity is $\mu\sqrt{\frac{c-x}{x}}$. Prove that its acceleration is directed towards O and is inversely proportional to the square of the velocity.

(d) Find the CG of the area of the cardioid $r = a(1+\cos\theta)$.

(e) A particle of mass m falls from rest at a height h above the ground. Show that the sum of its potential and kinetic energies is constant throughout the motion.

(f) A shot of mass m is fired from a gun of mass M with velocity u relative to the gun. Show that the actual velocities of the shot and the gun are $\frac{Mu}{M+m}$ and $\frac{mu}{M+m}$ respectively.

(g) A uniform ladder rests in limiting equilibrium with lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical, then prove that $\tan\theta = 2\mu$, where μ is the coefficient of friction.

(h) A particle of mass m is projected vertically upwards under gravity, the resistance of air being mk times the velocity. Show that the greatest height attained by the particle is

$$\frac{V^2}{g}[\lambda - \log(1+\lambda)]$$

where V is the terminal velocity of the particle and λV is the initial velocity.

4. Answer the following questions (any *two*) : 10×2=20

(a) State and prove Lami's theorem.

Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along $\vec{IA}$, $\vec{IB}$, $\vec{IC}$, where I is the incentre of the triangle ABC, are in equilibrium. Show that

$$P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$

1+4+5=10

(b) Prove that in polar coordinates (r, θ), the velocity $\vec{V}$ and acceleration $\vec{a}$ of a particle moving in a plane curve are given by

$$\vec{V} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

10

(c) (i) Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x\cos\theta + y\sin\theta = p$. Find the magnitude of the resultant and equation of its line of action. 5

(ii) A heavy uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and being

placed upon a peg at a distance b from the wall. Show that the inclination of the rod to the horizon θ is given by $\cos^3\theta = \frac{b}{a}$. 5

(d) (i) Find the centre of gravity of the solid formed by revolution of the quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about its minor axis. 5

(ii) A particle falls under gravity in a resisting medium whose resistance varies as the square of the velocity. Find the velocity of the particle at time t. 5

(e) (i) A particle is describing a plane curve. If the tangential and normal accelerations are each constant throughout the motion, then prove that the angle ψ, through which the direction of motion turns in time t, is given by $\psi = A\log(1 + Bt)$. 4

(ii) A particle of mass m is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin O. If it starts from rest at a distance a from O, then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 6

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