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3 (Sem-5/CBCS) MAT HC 1 (N/O)

2025

MATHEMATICS

(Honours Core)

Paper : MAT-HC-5016

(For New Syllabus)

(Complex Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any seven** questions from the following as directed : $1 \times 7 = 7$

(a) If z_0 is a point in the z -plane, then

$$\lim_{z \rightarrow \infty} f(z) = \infty \text{ if}$$

$$(i) \quad \lim_{z \rightarrow 0} \frac{1}{f(z)} = \infty$$

$$(ii) \quad \lim_{z \rightarrow 0} f\left(\frac{1}{z}\right) = 0$$

$$(iii) \lim_{z \rightarrow 0} \frac{1}{f(z)} = 0$$

$$(iv) \lim_{z \rightarrow 0} \frac{1}{f\left(\frac{1}{z}\right)} = 0$$

(Choose the correct answer)

(b) If $\cos z = 0$, then

$$(i) z = n\pi, (n = 0, \pm 1, \pm 2, \dots)$$

$$(ii) z = \frac{\pi}{2} + n\pi, (n = 0, \pm 1, \pm 2, \dots)$$

$$(iii) z = 2n\pi, (n = 0, \pm 1, \pm 2, \dots)$$

$$(iv) z = \frac{\pi}{2} + 2n\pi, (n = 0, \pm 1, \pm 2, \dots)$$

(Choose the correct answer)

(c) If $z = x + iy$, then $\sin z$ is

$$(i) \sin x \cosh y + i \cos x \sinh y$$

$$(ii) \cos x \cosh y - i \sin x \sinh y$$

$$(iii) \cos x \sinh y + i \sin x \cosh y$$

$$(iv) \sin x \sinh y - i \cos x \cosh y$$

(Choose the correct answer)

(d) The value of $\log(-1)$ is

$$(i) 0$$

$$(ii) 2n\pi i$$

$$(iii) \pi i$$

$$(iv) -\pi i \quad (\text{Choose the correct answer})$$

(e) $\exp(2 \pm 3\pi i)$ is

$$(i) -e^2$$

$$(ii) e^2$$

$$(iii) 2e$$

$$(iv) -2e \quad (\text{Choose the correct answer})$$

(f) Determine the singular points of the

$$\text{function } f(z) = \frac{2z+1}{z(z^2+1)}.$$

(g) When a function f is said to be analytic at a point?

(h) Write the Cauchy-Riemann equations for a function $f(z) = u + iv$.

(i) Determine the accumulation points of the set $Z_n = \frac{i}{n} (n = 1, 2, 3, \dots)$.

(j) Verify that $(3, 1) (3, -1) \left(\frac{1}{5}, \frac{1}{10}\right) = (2, 1)$.

(k) What is the multiplicative inverse of a non-zero complex number $z = (x, y)$?

(l) Describe the domain of definition of the function $f(z) = \frac{z}{z + \bar{z}}$.

2. Answer **any four** questions from the following : 2×4=8

(a) Show that $\int_0^{\frac{\pi}{6}} e^{i2t} dt = \frac{\sqrt{3}}{4} + \frac{i}{4}$.

(b) State the Cauchy integral formula. Find

$$\frac{1}{2\pi i} \int_C \frac{1}{z - z_0} dz \text{ if } z_0 \text{ is any point}$$

interior to simple closed contour C.

(c) If z_0 and w_0 are points in the z -plane and w -plane respectively, then prove that $\lim_{z \rightarrow z_0} f(z) = \infty$ if and only if

$$\lim_{z \rightarrow z_0} \frac{1}{f(z)} = 0.$$

(d) Show that

$$2 \sin(z_1 + z_2) \sin(z_1 - z_2) = \cos 2z_2 - \cos 2z_1.$$

(e) Show that $\log(i^3) \neq 3 \log i$.

(f) Find all values of z such that $\exp(2z - 1) = 1$.

(g) Define a connected set and give *one* example.

(h) Reduce the quantity $\frac{5i}{(1-i)(2-i)(3-i)}$ to a real number.

3. Answer **any three** questions from the following : 5×3=15

(a) Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$.

(b) Prove that any polynomial $P(z) = a_0 + a_1z + a_2z^2 + \dots + a_nz^n$ ($a_n \neq 0$) of degree n ($n \geq 1$) has at least one zero.

(c) Evaluate : $2\frac{1}{2}+2\frac{1}{2}=5$

(i)
$$\int_C \frac{e^{-z}}{z-(\pi i/2)} dz$$

(ii)
$$\int_C \frac{z}{2z+1} dz$$

where C denotes the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$.

(d) If a function f is analytic at all points interior to and on a simple closed contour C , then prove that

$$\int_C f(z) dz = 0.$$

(e) (i) Show that the function $f(z) = \exp \bar{z}$ is not analytic anywhere. 3

(ii) Find all roots of the equation

$$\log z = i \frac{\pi}{2}. \quad 2$$

(f) (i) Show that for the function $f(z) = \bar{z}$, $f'(z)$ does not exist anywhere. 3

(ii) Show that $\lim_{z \rightarrow \infty} \frac{4z^2}{(z-1)^2} = 4$. 2

(g) Suppose that $\lim_{z \rightarrow z_0} f(z) = w_0$ and

$$\lim_{z \rightarrow z_0} F(z) = W_0.$$

Prove that $\lim_{z \rightarrow z_0} [f(z)F(z)] = w_0W_0$.

(h) (i) If a and b are complex constants, use definition of limit to show that

$$\lim_{z \rightarrow z_0} (az + b) = az_0 + b. \quad 2$$

(ii) Show that

$$\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}} \right)^2 \text{ does not exist.} \quad 3$$

4. Answer **any three** questions from the following : 10×3=30

(a) (i) Suppose that $z_n = x_n + iy_n$, ($n=1, 2, \dots$) and $z = x + iy$. Prove that $\lim_{n \rightarrow \infty} z_n = z$ if and only if

$$\lim_{n \rightarrow \infty} x_n = x \text{ and } \lim_{n \rightarrow \infty} y_n = y. \quad 5$$

(ii) Show that

$$z^2 e^{3z} = \sum_{n=2}^{\infty} \frac{3^{n-2}}{(n-2)!} z^n \quad (|z| < \infty). \quad 5$$

(b) (i) Apply the Cauchy-Goursat theorem to show that $\int_C f(z) dz = 0$ when the contour C is the unit circle $|z|=1$, in either direction and $f(z) = ze^{-z}$. 4

(ii) If C is the positively oriented unit circle $|z|=1$ and $f(z) = \exp(2z)$, find $\int_C \frac{f(z)}{z^4} dz$. 3

(iii) Let z_0 be any point interior to a positively oriented simple closed curve C . Show that

$$\int_C \frac{dz}{(z - z_0)^{n+1}} = 0, \quad (n = 1, 2, \dots). \quad 3$$

(c) (i) Evaluate $I = \int_C z^{a-1} dz$

where C is the positively oriented circle $z = Re^{i\theta}$ ($-\pi \leq \theta \leq \pi$) about the origin and a denote any non-zero real number.

If a is a non-zero integer n , then what is the value of $\int_C z^{n-1} dz$?

$$4+1=5$$

(ii) Let C denote a contour of length L , and suppose that a function $f(z)$ is piecewise continuous on C . If μ is a non-negative constant such that $|f(z)| \leq \mu$ for all point z on C at which $f(z)$ is defined, then prove

$$\text{that } \left| \int_C f(z) dz \right| \leq \mu L.$$

Use it to show that $\left| \int_C \frac{dz}{z^2 - 1} \right| \leq \frac{\pi}{3}$

where C is the arc of the circle $|z|=2$ from $z=2$ to $z=2i$ that lies in the 1st quadrant. 3+2=5

(d) (i) Prove that $\int_C \frac{dz}{z} = \pi i$ where C is

the right-hand half $z = 2e^{i\theta}$
 $\left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$ of the circle $|z| = 2$

from $z = -2i$ to $z = 2i$. 5

(ii) If a function f is analytic everywhere inside and on a simple closed contour C , taken in the positive sense, then prove that

$$f'(z) = \frac{1}{2\pi i} \int_C \frac{f(s)}{(s-z)^2} ds \text{ where } s$$

denotes points on C and z is interior to C . 5

(e) (i) Prove that the existence of the derivative of a function at a point implies the continuity of the function at that point.

With the help of an example show that the continuity of a function at a point does not imply the existence of derivative there.

3+5=8

(ii) Find $f'(z)$ if

$$f(z) = \frac{z-1}{2z+1} \left(z \neq -\frac{1}{2}\right). \quad 2$$

(f) Let the function

$f(z) = u(x, y) + iv(x, y)$ be defined throughout some ε -neighbourhood of a point $z_0 = x_0 + iy_0$. If u_x, u_y, v_x, v_y exist everywhere in the neighbourhood, and these partial derivatives are continuous at (x_0, y_0) and satisfy the Cauchy-Riemann equations at (x_0, y_0) , then prove that $f'(z_0)$ exist and $f'(z_0) = u_x + iv_x$ where the right hand side is to be evaluated at (x_0, y_0) .

Use it to show that for the function

$$f(z) = e^{-x} \cdot e^{-y}, f''(z) \text{ exists everywhere}$$

$$\text{and } f''(z) = f(z). \quad 6+4=10$$

(g) (i) Suppose that $f(z_0) = g(z_0) = 0$ and that $f'(z_0)$ $g'(z_0)$ exist, where $g'(z_0) \neq 0$. Use definition of derivative to show that

$$\lim_{z \rightarrow z_0} \frac{f(z)}{g(z)} = \frac{f'(z_0)}{g'(z_0)}. \quad 3$$

(ii) Show that $f'(z)$ does not exist at any point if $f(z) = 2x + ixy^2$.

3

(iii) If a function f is analytic at a given point, then prove that its derivatives of all orders are analytic there too.

4

(h) (i) If a function f is continuous throughout a region R that is both closed and bounded, then prove that there exists a non-negative real number μ such that $|f(z)| \leq \mu$ for all points z in R , where equality holds for at least one such z .

4

(ii) Let a function $f(z) = u(x, y) + iv(x, y)$ be analytic throughout a given domain D . If $|f(z)|$ is constant throughout D , then prove that $f(z)$ must be constant there too.

3

(iii) Show that the function $f(z) = \sin x \cosh y + i \cos x \sinh y$ is entire.

3

(For Old Syllabus)

(Riemann Integration and Metric Spaces)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :

1×10=10

(a) Define the upper and the lower Darboux sums of a function $f : [a, b] \rightarrow \mathbb{R}$ with respect to a partition P .

(b) Prove that $\sqrt{\alpha+1} = \alpha \sqrt{\alpha}$.

(c) What is meant by partition P of an interval $[a, b]$?

(d) Let (X, d) and (Y, ρ) be metric spaces and $A \subset X$. Let $f : X \rightarrow Y$ be continuous on X . Then

(i) $f(A) = f(\overline{A})$

(ii) $f(\overline{A}) \subset \overline{f(A)}$

(iii) $\overline{f(A)} \subset f(\overline{A})$

(iv) $f(A) = f(A^0)$

(Choose the correct answer)

(e) Let $\{F_n\}$ be a decreasing sequence of closed subsets of a complete metric space and $d(F_n) \rightarrow 0$ as $n \rightarrow \infty$. Then

(i) $\bigcap_{n=1}^{\infty} F_n = \phi$

(ii) $\bigcap_{n=1}^{\infty} F_n$ contains at least one point

(iii) $\bigcap_{n=1}^{\infty} F_n$ contains exactly one point

(iv) $d\left(\bigcap_{n=1}^{\infty} F_n\right) > 0$

(Choose the correct answer)

(f) In a complete metric space

(i) every sequence is bounded

(ii) every bounded sequence is convergent

(iii) every convergent sequence is bounded

(iv) every Cauchy sequence is convergent

(Choose the correct answer)

(g) Let A and B be two subsets of a metric space (X, d) . Then

(i) $(A \cap B)^0 = A^0 \cap B^0$

(ii) $(A \cup B)^0 = A^0 \cup B^0$

(iii) $(A \cap B)' = A' \cap B'$

(iv) $(A \cup B)' = A' \cup B'$

where A^0 denotes interior of A

A' denotes derived set of A

(Choose the correct answer)

(h) Define Cauchy sequence in a metric space (X, d) .

(i) Find the limit point of the set

$$\left\{ 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots \right\}.$$

(j) Describe an open ball on the real line \mathbb{R} for the usual metric d .

2. Answer the following questions : $2 \times 5 = 10$

(a) Examine the existence of improper

integral $\int_0^1 \frac{1}{\sqrt{x}} dx$.

(b) Find $L(f, P)$ and $U(f, P)$ for a constant function $f : [a, b] \rightarrow \mathbb{R}$.

(c) Let (X, d_X) and (Y, d_Y) be metric spaces and let $f : X \rightarrow Y$. Then if f is continuous on X , prove that $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B})$ for all subsets B of Y .

(d) For any two subsets F_1 and F_2 of a metric space (X, d) , prove that

$$\overline{(F_1 \cup F_2)} = \overline{F_1} \cup \overline{F_2}.$$

(e) Prove that in a discrete metric space every singleton set is open.

3. Answer **any four** parts : $5 \times 4 = 20$

(a) Let $f : [a, b] \rightarrow \mathbb{R}$ be bounded and monotone. Prove that f is integrable.

(b) Consider the function $f : [0, 1] \rightarrow \mathbb{R}$

defined by $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{otherwise} \end{cases}$

Prove that f is not integrable on $[0, 1]$.

(c) Let (X, d) be a metric space and let $\{Y_\lambda, \lambda \in I\}$ be a family of connected sets in (X, d) having a non-empty intersection. Then prove that $Y = \bigcup_{\lambda \in I} Y_\lambda$

is connected.

(d) Prove that if a Cauchy sequence of points in a metric space (X, d) contains a convergent subsequence, then the sequence also converges to the same limit as the subsequence.

(e) In a metric space (X, d) , prove that every closed sphere is a closed set.

(f) Let d be a metric on the non-empty set X . Prove that the function d' defined by $d'(x, y) = \min\{1, d(x, y)\}$

where $x, y \in X$ is a metric on X . State whether d' is bounded or not.

4+1=5

4. Answer **any four** parts : 10×4=40

(a) (i) Prove that a mapping $f : X \rightarrow Y$ is continuous on X if and only if $f^{-1}(F)$ is closed in X for all closed subsets F of Y . 5

(ii) Let f and g be continuous on $[a, b]$. Also assume that g does not change sign on $[a, b]$. Then prove that for some $c \in [a, b]$ we have

$$\int_a^b f(x) g(x) dx = f(c) \int_a^b g(x) dx.$$

5

(b) Let $f : [0, 1] \rightarrow \mathbb{R}$ be continuous. Let

$C_i \in \left[\frac{i-1}{n}, \frac{i}{n} \right], n \in \mathbb{N}$. Then prove that

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(C_i) = \int_0^1 f(x) dx.$$

Using it, prove that $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{k^2 + n^2} = \frac{\pi}{4}$.

6+4=10

(c) Prove that a bounded function $f : [a, b] \rightarrow \mathbb{R}$ is integrable if and only if for each $\varepsilon > 0$, there exists a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

(d) (i) Show that the metric space (X, d) where X denotes the space of all sequences $x = (x_1, x_2, x_3, \dots, x_n)$ of real numbers for which

$$\left(\sum_{k=1}^{\infty} |x_k|^p \right)^{\frac{1}{p}} < \infty \quad (p \geq 1) \text{ and } d \text{ is}$$

the metric given by

$$d_p(x, y) = \left(\sum_{k=1}^{\infty} (x_k - y_k)^p \right)^{1/p}, \quad x, y \in X$$

is a complete metric space. 7

(ii) Let X be any non-empty set and let d be defined by

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

Show that (X, d) is a complete metric space. 3

(e) When a metric space is said to be disconnected?

Prove that a metric space (X, d) is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed in (X, d) .

2+8=10

(f) (i) Prove that the function $f : [0, 1] \rightarrow \mathbb{R}$ defined by

$$f(x) = x^2 \text{ is uniformly}$$

continuous. Further prove that the function will not be uniformly continuous if the domain is \mathbb{R} .

3+3=6

- (ii) Let (X, d_X) , (Y, d_Y) and (Z, d_Z) be metric spaces and let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be continuous. Prove that the composition $g \circ f$ is a continuous map of X into Z .
4

(g) Let Y be a subspace of a metric space (X, d) . Prove the following : 5+5=10

- (i) Every subset of Y that is open in Y is also open in X if and only if Y is open in X .
- (ii) Every subset of Y that is closed in Y is also closed in X if and only if Y is closed in X .

(h) (i) Define a metric space.

Let

$$X = \mathbb{R}^n = \{x = (x_1, x_2, \dots, x_n), x_i \in \mathbb{R}, 1 \leq i \leq n\}$$

be the set of all real n -tuples.

For $x = (x_1, x_2, \dots, x_n)$ and

$y = (y_1, y_2, \dots, y_n)$ in \mathbb{R}^n define

$$d(x, y) = \left(\sum_{i=1}^n (x_i - y_i)^2 \right)^{1/2}.$$

Prove that (\mathbb{R}^n, d) is a metric space.
2+4=6

- (ii) Prove that in a metric space (X, d) , a finite intersection of open sets is open.
4

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3 (Sem-5 /CBCS) MAT HC 2

2025

MATHEMATICS

(Honours Core)

Paper : MAT-HC-5026

(Linear Algebra)

Full Marks : 80

Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

1. Answer the following as directed : $1 \times 10 = 10$
- (i) What do you mean by the length of a vector in \mathbb{R}^n ?
- (ii) How do you explain $\dim W = 1$ geometrically where W is a subspace of the vector space $\mathbb{R}^3(\mathbb{R})$?

(iii) Let A be the 4×4 real matrix,

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -2 & -2 & 0 \\ 1 & 1 & -2 & 1 \end{bmatrix}$$

then the characteristic polynomial for A is

(A) $x^2(x-1)^2$

(B) $(x-1)^2(x+1)^2$

(C) $x^2(x+1)^2$

(D) None of the above

(Choose the correct option)

(iv) Which vector is orthogonal to every vector in \mathbb{R}^n ?

(v) Define inner product on \mathbb{R}^n .

(vi) "The eigenvalues of a triangular matrix are the entries on its main diagonal." (State True or False)

(vii) Write the standard basis of the vector space of polynomial in x with real coefficient of degree ≤ 3 .

(viii) Let S be a subset of a vector space $V(F)$ and S contains zero vector of V . Then S is

(A) linearly independent

(B) linearly dependent

(C) Both linearly independent and linearly dependent

(D) None of the above

(Choose the correct option)

(ix) Let A be a 5×4 matrix. If null space of A is a subspace of \mathbb{R}^k then what is k ?

(x) Is $\mathbb{R}^2(\mathbb{R})$ is a subspace of $\mathbb{R}^3(\mathbb{R})$?

2. Answer the following questions : $2 \times 5 = 10$

(i) Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y onto u .

(ii) Let T be the linear operator on \mathbb{R}^3 defined by $T(a, b, c) = (a + b, b + c, 0)$. Show that the

xy -plane = $\{(x, y, 0) : x, y \in \mathbb{R}\}$ is T -invariant subspace of \mathbb{R}^3 .

(iii) Show that every subset of an independent set is independent.

(iv) Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Is v an eigenvector of A ?

(v) Let V be the vector space of all functions from the real field \mathbb{R} to \mathbb{R} . Show that $W = \{f : f(7) = 2 + f(1)\}$ is not a subspace of V .

3. Answer **any four** questions : $5 \times 4 = 20$

(i) Define orthogonal set. If $S = \{u_1, u_2, \dots, u_p\}$ is an orthogonal set of non-zero vectors in \mathbb{R}^n , then prove that S is linearly independent and hence is a basis for the subspace spanned by S .

(ii) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula

for A^2 , given that $A = PDP^{-1}$ where

$$P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \text{ and } D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}.$$

(iii) Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix}$. An eigenvalue of

A is 2. Find a basis for the corresponding eigenspace.

(iv) Let A and B be two similar matrices of order $n \times n$. Prove that A and B have same characteristic polynomial and hence the same eigenvalues.

(v) Let v_1, v_2, \dots, v_n be non-zero eigenvectors of an operator $T: V \rightarrow V$ corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$. Prove that v_1, v_2, \dots, v_n are linearly independent.

(vi) Prove that the non-zero vectors v_1, v_2, \dots, v_n are linearly dependent if and only if one of them is a linear combination of the preceding vectors.

4. (i) Define orthonormal set and orthonormal basis in \mathbb{R}^n . Show that $\{u_1, u_2, u_3\}$ is an orthonormal basis of \mathbb{R}^3 , where

$$u_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix}, \quad u_2 = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \quad u_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}.$$

$$1+1+8=10$$

OR

Define inner product space. Show that the following is an inner product in \mathbb{R}^2 :

$$\langle u, v \rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2,$$

where $u = (x_1, x_2)$, $v = (y_1, y_2)$.

Also show that for all u, v in \mathbb{R}^2

$$\|u+v\| \leq \|u\| + \|v\|.$$

$$2+5+3=10$$

- (ii) Let T be the linear operator on \mathbb{R}^3 , defined by

$$T(x, y, z) = (2y+z, x-4y, 3x).$$

- (a) Find the matrix of T in the basis

$$\{e_1 = (1, 1, 1), e_2 = (1, 1, 0), e_3 = (1, 0, 0)\}$$

- (b) Verify that $[T]_e[v]_e = [T(v)]_e$ for any vector $v \in \mathbb{R}^3$. 4+6=10

OR

An $n \times n$ matrix A is diagonalizable if and only if A has n linearly independent eigenvectors. 10

- (iii) Find the eigenvalues and eigenvectors

$$\text{of } A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

10

OR

State Cayley-Hamilton theorem for matrices. Use it to express

$$2A^5 - 3A^4 - A^2 - 4I$$

as a linear polynomial in A , when $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

- (iv) If a vector space V has a basis $B = \{v_1, v_2, \dots, v_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent. Also show that every basis of V must consist of exactly n vectors. 5+5=10

OR

Let U and V be vector spaces over the same field. Let $\{u_1, u_2, \dots, u_n\}$ be a basis of U and let v_1, v_2, \dots, v_n be any arbitrary vectors in V . Prove that there exists a unique linear mapping $f: U \rightarrow V$ such that

$$f(u_1) = v_1, f(u_2) = v_2, \dots, f(u_n) = v_n. \quad 10$$
