

1 (Sem-5/FYUGP) MAT 43 MJ

2025

MATHEMATICS

( Major )

Paper : MAT0500304

( Numerical Analysis—I )

Full Marks : 45

Time : 2 hours

*The figures in the margin indicate full marks  
for the questions.*

1. Answer the following as directed : 1×5=5

(a) In the Gaussian elimination method, the coefficient matrix of the linear system of equations reduces to

(i) upper triangular matrix

(ii) diagonal matrix

(iii) lower triangular matrix

(iv) None of the above

(Choose the correct option)

(b) Which of the following is an iterative method?

(i) Gauss-Jordan method

(ii) Lagrange's method

(iii) Gauss-Seidel method

(iv) Newton's divided difference method  
(Choose the correct option)

(c) Evaluate  $\Delta^2(3e^x)$ .

(d) What is numerical integration? Mention one method of numerical integration.

(e) In Simpson's  $\frac{1}{3}$ rd rule, the degree of interpolating polynomial is

(i) 1

(ii) 3

(iii) 2

(iv) 5 (Choose the correct option)

2. Answer any five of the following questions :

$$2 \times 5 = 10$$

(a) Prove the operator relation

$$(1 + \Delta)(1 - \nabla) \equiv 1$$

(b) Given  $u_0 = 580$ ,  $u_1 = 556$ ,  $u_2 = 520$ ,  $u_4 = 385$ , find the value of  $u_3$ .

(c) A second-degree polynomial passes through (0, 1), (1, 3), (2, 7) and (3, 13). Find the polynomial.

(d) Construct a divided difference table for the following data :

$x$	:	0	2	3	4	7	9
$f(x)$	:	4	26	58	112	466	922

(e) Consider the following system of equations :

$$4x_1 + x_2 + x_3 = 2$$

$$x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 3x_3 = -4$$

To use Jacobi iteration method

$$x^{(k+1)} = Hx^{(k)} + c$$

for solving this system, find out the coefficient matrix  $H$ .

(f) Write a short note on piecewise linear interpolation method.

(g) What is Richardson's extrapolation? Write briefly.

(h) Write the formulae for first- and second-order derivatives of  $y = f(x)$  using Newton's forward difference interpolation formula.

(i) Write the general scheme of modified Euler's method.

(j) If  $M(h) = \frac{1}{2h} [-y(x+2h) + 4y(x+h) - 3y(x)]$ , show that

$$y'(x) - M(h) = c_1 h^2 + c_2 h^3 + c_3 h^4 + \dots$$

where  $c_1, c_2, c_3, \dots$  are constants independent of  $h$ .

3. Answer any *four* of the following questions :

5×4=20

(a) Find the number of students from the following data who secured marks not more than 45 :

Marks	:	30-40	40-50	50-60	60-70	70-80
No. of students	:	35	48	70	40	22

(b) Show that the  $n$ th divided difference of a polynomial of degree  $n$  is constant.

(c) Solve the following system of equations using Gauss elimination method :

$$2x_1 + 4x_2 + x_3 = 3$$

$$3x_1 + 2x_2 - 2x_3 = 2$$

$$x_1 - x_2 + x_3 = 6$$

(d) Construct the Gauss-Seidel scheme for the system

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1$$

(e) Find  $f'(4)$  from the following table :

$x$	:	1	2	4	8	10
$f(x)$	:	0	1	5	21	27

(f) Evaluate

$$\int_{0.5}^{0.7} \sqrt{x} e^{-x} dx$$

by Simpson's  $\frac{1}{3}$ rd rule dividing the interval into four equal parts.

(g) Calculate the approximate value of

$$\int_{-3}^3 x^4 dx$$

using trapezoidal rule dividing the interval into six equal parts. Compare with the exact value.

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- (h) If  $\frac{dy}{dx} = x^2 + y^2$  with  $y(0) = 0$ , find  $y(0.3)$  using Euler's method with  $h = 0.1$ .

4. Answer any one of the following questions : 10

- (a) Let  $y = f(x)$  be a function of the independent variable  $x$  and  $y_0, y_1, \dots, y_n$  be the values of  $y$  corresponding to  $x_0, x_1, \dots, x_n$ . Derive Lagrange's interpolation formula to find the value of  $y$  at any point  $x$  between  $x_0$  and  $x_n$ . Use it to find  $f(4)$  from the following table :

$$5+5=10$$

$x$	:	0	1	2	5
$f(x)$	:	2	5	7	8

- (b) Obtain the piecewise quadratic interpolating polynomial for the function  $f(x)$  defined by the following data :

$x$	:	-2	-1	1	2	4
$f(x)$	:	-29	-8	-2	-5	7

- (c) Evaluate  $\int_1^2 e^x dx$  using trapezoidal rule considering 2, 3, 5, 9 nodes and Romberg integration method.

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- (d) Solve the initial value problem

$$\frac{dy}{dx} = -2xy^2, \quad y(0) = 1$$

using the mid-point method with  $h = 0.2$  over the interval  $[0, 1]$ .

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2025

MATHEMATICS

( Major )

Paper : MAT0500404

( Abstract Algebra )

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks  
for the questions.

1. Answer the following : 1×8=8

- (a) State the Lagrange's theorem on order of subgroups of a group.
- (b) If  $H$  and  $K$  are two finite subgroups of a group, then which of the following is true?

(i)  $O(HK) = \frac{O(H) + O(K)}{O(H \cap K)}$

(ii)  $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$

(iii)  $O(HK) = O(H) + O(K)$

(iv)  $O(HK) = O(H)O(K)$

- (c) Write True or False :  
"Order of a cyclic group is equal to the order of its generators."
- (d) Give an example of a left ideal which is not a right ideal.

(e) Express the permutation

$$\begin{pmatrix} a & b & c & d & e & f & g \\ c & d & e & g & f & b & a \end{pmatrix}$$

as a cycle.

(f) Under what condition,

$$Z_p = \{0, 1, 2, \dots, (p-1)\}$$

modulo  $p$  will be a field?

(g) When will an element in a ring be called a nilpotent element?

(h) Give an example of a prime ideal of a ring which is not a maximal ideal in that ring.

( Continued )

2. Answer any six from the following :  $2 \times 6 = 12$

(a) Let  $G$  be a group and  $a$  be any element of  $G$ . Show that  $\langle a \rangle$ , the subset generated by  $a$ , is a subgroup of  $G$ .

(b) Define a ring homomorphism and its kernel.

(c) Show that every subgroup of a cyclic group is a cyclic group.

(d) Define the centre of a group and calculate the centre of  $S_3$ .

(e) Show that the centre of any group  $G$  is a normal subgroup of  $G$ .

(f) If  $G = Z(G)$  is cyclic, then show that  $G$  is Abelian.

(g) If  $Z$  is the ring of integers and  $H = \{3n : n \in Z\}$ , then write all right cosets of  $H$  in  $Z$ .

(h) If  $x$  is an element of a group and  $x^n = e$  the identity element of the group, then show that the order of  $x$  divides  $n$ .

(i) Show that every ideal of the ring of integers is a principal ideal.

(j) If  $f: R \rightarrow S$  is a ring homomorphism, then show that kernel of  $f$  is an ideal of  $R$ .

3. Answer any four from the following :  $5 \times 4 = 20$

(a) If  $f$  is a group homomorphism from  $G$  onto  $H$ , then show that  $H \cong G/K$ , where  $K$  is the kernel of  $f$ .

(b) Describe in pictures the elements of the dihedral group  $D_4$  of the symmetries of a square.

(c) Write all subgroups of  $Z_{30}$ .

(d) Suppose  $f: G \rightarrow H$  is a group homomorphism, then prove the following :  $2+2+1=5$

(i) If  $e$  is the identity of  $G$ , then  $f(e)$  is the identity of  $H$ .

(ii) If  $K$  is a subgroup of  $G$ , then  $f(K)$  is a subgroup of  $H$ .

(iii) If  $a$  is an element of  $G$ , then  $f(a^{-1}) = (f(a))^{-1}$ .

(e) Define a transposition in permutation. Show that every permutation can be expressed as a product of transpositions.

(f) If  $D$  is a commutative ring without zero-divisors, show that the characteristic of  $D$  is either a zero or a prime number.

(g) If  $P$  and  $Q$  are two ideals of a ring  $R$ , then show that  $P+Q$  is again an ideal of  $R$  containing each of  $P$  and  $Q$ .

(h) Define zero-divisor in a ring. Show that a finite integral domain is a field. Give an example of an integral domain which is not a field.  $1+3+1=5$

4. Answer any two from the following :  $10 \times 2 = 20$

(a) (i) Define a quotient group. Show that every quotient group of a cyclic group is cyclic. 5

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- (ii) Show that a group of prime order has no non-trivial subgroup. 2
- (iii) If  $a$  is an integer and  $p$  is a prime, show that  $a^p \equiv a \pmod{p}$ . 3
- (b) (i) Define even and odd permutations. Show that a cycle of even length is an odd permutation and a cycle of odd length is an even permutation. 2+3=5
- (ii) Compute  $a^{-1}bab^{-1}$  where  $a = (135)(14)$  and  $b = (2576)$ . 4
- (iii) Find the generators of the group  $\{1, -1, i, -i\}$  with multiplication. 1
- (c) Show that every group is isomorphic to a permutation group.
- (d) If  $R$  is a commutative ring with unity, then show that an ideal  $M$  will be a maximal ideal in  $R$  if and only if  $R/M$  is a field.

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- (e) (i) Prove that a group homomorphism  $f$  will be one-one if and only if the kernel of  $f$  contains only the identity element. 5
- (ii) Prove that a subgroup of index 2 in a group is a normal subgroup. 5

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