1 (Sem–3/FYUGP) MAT41 MJ

2025

MATHEMATICS

(Major)

Paper : MAT4300104MJ

(Ordinary Differential Equations)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

*Answer **either** in English **or** in Assamese.*

1. Answer the following questions : 1×8=8

    তলত দিয়া প্রশ্নবোৰৰ উত্তৰ কৰা :

    . *(a)* How many independent variables are involved in an ordinary differential equation ?

    এটা সাধাৰণ অৱকল সমীকৰণত কিমানটা স্বতন্ত্র চলক যুক্ত হৈ থাকে ?

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(b) Consider the equation (তলৰ সমীকৰণটো লোৱা) $(x+2y+3)dx+(2x+4y-1)dy=0$. Can this equation be reduced to a homogeneous equation?

এই সমীকৰণটো এটা সমমাত্ৰিক সমীকৰণলৈ হ্ৰাস কৰিব পাৰি নে?

(c) Find the integrating factor of the linear differential equation $x^4\dfrac{dy}{dx}+2x^3y=1$.

ৰৈখিক অৱকল সমীকৰণ $x^4\dfrac{dy}{dx}+2x^3y=1$ ৰ অনুকলন গুণক উলিওৱা।

(d) Define Wronskian of the real functions $f_1, f_2 \ldots \ldots f_n$ in $a \le x \le b$.

$a \le x \le b$ অন্তৰালত বাস্তৱ ফলন $f_1, f_2 \ldots \ldots f_n$ ৰ Wronskian ৰ সংজ্ঞা দিয়া।

(e) Find the general solution of $\dfrac{d^2y}{dx^2}+y=0$.

$\dfrac{d^2y}{dx^2}+y=0$ ৰ সাধাৰণ সমাধান উলিওৱা।

(f) State one difference between complementary function and particular integral related to solutions of differential equations.

অৱকল সমীকৰণৰ সমাধানৰ লগত যুক্ত পূৰক ফলন আৰু বিশেষ অনুকলৰ মাজৰ এটা পাৰ্থক্য লিখা।

(g) Can method of undetermined co-efficients be applied to find particular integral of $\dfrac{d^2y}{dx^2}+y=\cot ax$? Give reasons.

$\dfrac{d^2y}{dx^2}+y=\cot ax$ ৰ বিশেষ অনুকল নিৰ্ণয় কৰিবলৈ অনিৰ্ণেয় সহগ পদ্ধতি প্ৰয়োগ কৰিব পাৰি নে? যুক্তি দিয়া।

(h) Examine whether the functions x^2 and $\dfrac{1}{x}$ are linearly independent or not.

x^2 আৰু $\dfrac{1}{x}$ ফলন দুটা ৰৈখিক ভাৱে স্বতন্ত্ৰ হয় নে পৰীক্ষা কৰা।

2. Answer **_any six_** of the following questions : 2×6=12

তলত দিয়া প্রশ্নবোৰৰ *যিকোনো ছয়টাৰ* উত্তৰ কৰা :

(a) What is meant by implicit and explicit solution of a differential equation ?

এটা অৱকল সমীকৰণৰ অস্পষ্ট বা পৰোক্ষ আৰু স্পষ্ট বা প্ৰত্যক্ষ সমাধান মানে কি বুজায় ?

(b) Find the most general function $N(x, y)$ such that the differential equation

$$\left(x^3 + xy^2\right)dx + N\left(x,y\right)dy = 0 \text{ is exact.}$$

এটা অতি সাধাৰণ ফলন $N(x, y)$ উলিওৱা যাতে

$$\left(x^3 + xy^2\right)dx + N\left(x,y\right)dy = 0 \text{ অৱকল}$$

সমীকৰণটো যথাৰ্থ হয়।

(c) Find the integrating factor of

অনুকলন গুণক উলিওৱা

$$\left(2x^2 + y\right)dx + \left(x^2y - x\right)dy = 0.$$

(d) Write the general form of an nth order Cauchy-Euler equation.

What transformation reduces it to a linear differential equation with constant co-efficients.

এটা n তম ঘাতৰ কচি-ইউলাৰ সমীকৰণৰ সাধাৰণ ৰূপটো লিখা।

কি ৰূপান্তৰে ইয়াক এটা ধ্ৰুৱক সহগ বিশিষ্ট ৰৈখিক সমীকৰণলৈ হ্ৰাস কৰে ?

(e) Show that $sin\ 3x$ and $cos\ 3x$ are linearly independent solutions of the differential equation $\dfrac{d^2y}{dx^2} + 9y = 0$.

দেখুওৱা যে $\dfrac{d^2y}{dx^2} + 9y = 0$ অৱকল সমীকৰণটোৰ

$sin\ 3x$ আৰু $cos\ 3x$ দুটা ৰৈখিক ভাৱে স্বতন্ত্ৰ সমাধান হয়।

(f) Explain briefly how a Bernoulli equation can be transformed to a linear equation.

এটা বাৰ্ণলী সমীকৰণ কেনেকৈ এটা ৰৈখিক সমীকৰণলৈ ৰূপান্তৰ কৰিব পাৰি চমুকৈ ব্যাখ্যা কৰা।

(g) The general solution of a second order homogeneous linear differential equation with constant co-efficients is $y = C_1 e^x + C_2 e^{2x}$. Construct the equation.

এটা দ্বিঘাতৰ ধ্ৰুৱক সহগ যুক্ত সমমাত্ৰিক ৰৈখিক অৱকল সমীকৰণৰ সাধাৰণ সমাধান হ'ল $y = C_1 e^x + C_2 e^{2x}$ সমীকৰণটো গঠন কৰা।

(h) Find the general solution :

সাধাৰণ সমাধান উলিওৱা ঃ

$$\frac{d^3y}{dx^3} + y = 0$$

(i) Find the particular integral of :

বিশেষ অনুকল উলিওৱা ঃ

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x} + 2$$

(j) Define total differential of a function $F(x, y)$.   1+1=2

Find total differential of

$$F(x,y) = xy^2 + 2x^3y.$$

$F(x, y)$ ফলনটোৰ পূৰ্ণ অৱকলৰ সংজ্ঞা দিয়া।

$F(x,y) = xy^2 + 2x^3y$ ফলনটোৰ পূৰ্ণ অৱকল উলিওৱা।

3. Answer **any four** of the following questions :   5×4=20

তলত দিয়া প্ৰশ্নবোৰৰ *যিকোনো চাৰিটাৰ উত্তৰ কৰা* ঃ

(a) Show that $x^3 + 3xy^2 = 1$ is an implicit solution of the differential equation $2xy\frac{dy}{dx} + x^2 + y^2 = 0$ on the interval $0 < x < 1$.

দেখুওৱা যে $0 < x < 1$ অন্তৰালত $2xy\frac{dy}{dx} + x^2 + y^2 = 0$ অৱকল সমীকৰণটোৰ $x^3 + 3xy^2 = 1$ এটা অন্তৰ্নিহিত সমাধান হয়।

(b) Solve the initial value problem

প্ৰাৰম্ভিক মানযুক্ত সমস্যাটো সমাধান কৰা

$$(y+2)dx + y(x+4y)dy = 0, \quad y(-3) = -1$$

(c) Solve by reducing to homogeneous equation.

সমমাত্ৰিক সমীকৰণলৈ হ্ৰাস কৰি সমাধান কৰা

$$(5x+2y+1)dx + (2x+y+1)dy = 0.$$

(d) Solve the Bernoulli equation

বাৰ্ণলী সমীকৰণটো সমাধান কৰা

$$\frac{dy}{dx} + \frac{2}{x}y = x^2 y^3.$$

(e) Given $y = x+1$ is a solution of the differential equation :

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0$$

Find a linearly independent solution by reducing the order.

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 3y = 0$$

অৱকল সমীকৰণটোৰ $y = x+1$ এটা সমাধান হয়। সমীকৰণটোৰ মাত্ৰা হ্ৰাস কৰি আনটো ৰৈখিক ভাৱে স্বতন্ত্ৰ সমাধান উলিওৱা।

(f) Solve (সমাধান কৰা) :

$$\frac{dy}{dx} + y = f(x) \quad \text{where (য'ত)}$$

$$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}, \quad y(0) = 0.$$

(g) Solve the initial value problem :

প্ৰাৰম্ভিক মানযুক্ত সমস্যাটো সমাধান কৰা ঃ

$$\frac{d^2y}{dx^2} - y = 4xe^x, \quad y(0) = 1, \, y'(0) = 2.$$

(h) Solve the Cauchy-Euler equation :

কচি - ইউলাৰ সমীকৰণটো সমাধান কৰা ঃ

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$$

4. Answer **any two** of the following questions : 10×2=20

তলত দিয়া প্ৰশ্নবোৰৰ *যিকোনো দুটাৰ* উত্তৰ কৰা ঃ

(a) Consider the differential equation

$$M(x, y)dx + N(x, y)dy = 0$$

Where M and N have continuous first partial derivatives at all points (x, y) in it's domain D.

Prove that the equation is exact in D if and only if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, for all $(x, y) \in D$

এটা অৱকল সমীকৰকণ লোৱা হ'ল

$$M(x, y)dx + N(x, y)dy = 0$$

য'ত M আৰু N দুয়োটা ফলনৰে আদিক্ষেত্ৰ D ৰ প্ৰতিটো বিন্দু (x, y) ত অবিছিন্ন আংশিক অৱকল থাকে।

প্ৰমাণ কৰা যে, সমীকৰণটো যথাৰ্থ হব যদি আৰু যদিহে

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \; \forall \; (x, y) \in D \text{ হয়।}$$

(b) (i) Prove that if f and g are two different solutions of 3

$$\frac{dy}{dx} + P(x)y = Q(x)$$

then $f - g$ is a solution of the equation

$$\frac{dy}{dx} + P(x)y = 0$$

দেখুওৱা যে $\dfrac{dy}{dx} + P(x)y = Q(x)$ অৱকল সমীকৰণটোৰ যদিহে f আৰু g দুটা সমাধান হয় তেন্তে $\dfrac{dy}{dx} + P(x)y = 0$ সমীকৰণটোৰ $f - g$ এটা সমাধান হব।

(ii) Consider the differential equation

$$\left(4x + 3y^2\right)dx + 2xy \, dy = 0$$

(I) Show that the equation is not exact.

(II) Find an integrating factor of the form x^n, where n is a positive integer.

(III) Multiply the given equation by the integrating factor found in (II) and solve the resulting exact equation.

$$1+2+4=7$$

তলৰ অৱকল সমীকৰণটো লোৱা।

$$\left(4x + 3y^2\right)dx + 2xy\,dy = 0$$

(I) দেখুওৱা যে সমীকৰণটো যথাৰ্থ ন হয়।

(II) এটা ধনাত্মক অখণ্ড সংখ্যা n ৰ বাবে সমীকৰণটোৰ x^n আহিব অনুকলন গুণকটো উলিওৱা।

(III) (II) ত প্ৰাপ্ত অনুকলন গুণকেৰে সমীকৰণটো পূৰণ কৰি লব্ধ যথাৰ্থ সমীকৰণটো সমাধান কৰা।

(c) Solve by method of variation of parameters

প্ৰাচলৰ তাৰতম্য পদ্ধতিৰে সমাধান কৰা

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}.$$

(d) Find the general solution

সাধাৰণ সমাধান উলিওৱা

$$\frac{d^4y}{dx^4} - 3\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} = 4\sin 2x + e^x.$$

(e) (i) Can the method of undetermined co-efficients be applied to find particular integral of all types of linear non-homogeneous differential equations with constant co-efficients ? Explain. 2

অনিৰ্ণেয় সহগ পদ্ধতি প্ৰয়োগ কৰি সকলোধৰণৰ ধ্ৰুৱক সহগ বিশিষ্ট বিসমমাত্ৰিক ৰৈখিক অৱকল সমীকৰণৰ বিশেষ অনুকল উলিওৱা সম্ভৱ নে? ব্যাখ্যা কৰা।

(ii) The particular integral assumed by the method of undetermined co-efficients for the differential equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^{4x}$

is $y_p = Ae^{4x}$

where as by the same method the particular integral assumed for the equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^{3x}$

is $y_p = A\,xe^{3x}$.

Explain the validity of the assumptions. 8

অনিৰ্ণেয় সহগ পদ্ধতি ব্যৱহাৰ কৰি

$$\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^{4x}$$

অৱকল সমীকৰণৰ বাবে ধৰি লোৱা বিশেষ অনুকল হৈছে $y_p = Ae^{4x}$

আনহাতে একেই পদ্ধতিৰে

$$\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 2e^{3x}$$

সমীকৰণৰ বাবে ধৰি লোৱা বিশেষ অনুকল হৈছে

$y_p = A\,xe^{3x}$

দুইটা ধাৰণাৰে বৈধতা ব্যাখ্যা কৰা।

1 (Sem–3/FYUGP) MAT42MJ

2025

MATHEMATICS

(Major)

Paper : MAT4300204 MJ

(Abstract Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions :   1×8=8

    (a) Define the symmetry group of a plane figure.

    (b) Under which condition the set $\{1, 2, 3, \ldots, n-1\}$, $n > 1$ form a group under multiplication modulo n.

(c) Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{pmatrix}$ in cyclic notation.

(d) Every permutation in S_n, $n > 1$, is a production of _____.

(Fill in the blank)

(e) Define isomorphism between two groups.

(f) Define $\phi : Z_{12} \to Z_{12}$ by $\phi(x) = 3x$, $x \in Z_{12}$. Find $\ker \phi$

(g) Give an example of a commulative ring with unity.

(h) What is the trivial ideal of any ring R?

2. Answer **any six** questions : 2×6=12

(a) Let G be a group. Show that $(ab)^{-1} = b^{-1}a^{-1}$, a, $b \in G$.

(b) Show that $H = \{x \in G : x \geq 1\}$ is not a subgroup of G, where G is the group of non-zero real numbers.

(c) Show that every cyclic group is abelian.

(d) Show that every permutation in S_n, $n > 1$ is a product of 2 – cycles.

(e) Show that every subgroup of an Abelian group is normal.

(f) Suppose ϕ is an isomorphism from a group G to a group $\overline{G}$. Then prove that, for $a \in G$, G=<a> if and only if $\overline{G} = \langle \phi(a) \rangle$.

(g) Let $\mathbb{R}^*$ be the group of nonzero real numbers under multiplication. Then prove that, the mapping $\phi : \mathbb{R}^* \to \mathbb{R}^*$ defined by $\phi(x) = |x|$, is a homomorphism and also find $\ker \phi$.

(h) Prove that $a(-b) = (-a)b = -ab$ for all a, b belongs to a ring $\mathbb{R}$.

(i) Show that $2Z \cup 3Z$ is not a subring of Z, Z the set of integers.

(j) If A and B are ideals of a ring, show that the sum

$A + B = \{a + b \mid a \in A, b \in B\}$, is an ideal.

3. Answer **any four** questions : 5×4=20

(a) H and K be subgroups of an abelian group G. Prove that HK is a subgroup of G.

(b) Show that $U(8)$ is not a cyclic group but $U(10)$ is cyclic.

(c) Prove that the set of all even permutation in S_n, forms a subgroup of S_n.

(d) Prove that for every integer a and every prime p, $a^p \equiv a \pmod{p}$.

(e) Suppose that ϕ is an isomorphism from a group G onto a group $\overline{G}$. If K is a subgroup of G, then prove that $\phi(K) = \{\phi(k) \mid k \in K\}$ is a subgroup of $\overline{G}$.

(f) Let ϕ be a homomorphism from a group G to a group $\overline{G}$ and let g be an element of G. If $\phi(g) = g'$, then prove that

$\phi^{-1}(g') = \{x \in G \mid \phi(x) = g'\} = g \ker \phi$.

(g) Show that the set

$\left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \mid a, b \in Z \right\}$ of diagonal matrices is a subring of the ring of all 2×2 matrices over Z.

(h) Prove that the characteristic of an integral domain is 0 or prime.

4. Answer **any two** questions :   10×2=20

(a) Prove that every subgroup of a cyclic group is cyclic. If $|\langle a\rangle|=n$, then show that the order of any subgroup of $\langle a\rangle$ is a divisor of n.   5+5=10

(b) Let G be a group and let H be a normal subgroup of G then prove that the set $G/H = \{aH \mid a \in G\}$ is a group under the operation $(aH)(bH) = abH$.

(c) Let H and K be two normal subgroups of a group G such that $H \subseteq K$. Show that $G/K \cong G/H \big/ K/H$.

(d) Prove that the set $Q[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a ring.

(e) Let R be a commutative ring with unity and let A be an ideal of R. Then prove that R/A is a field if and only if A is maximal.

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