1A (Sem–1/ITEP) MAT01 MJ

2025

MATHEMATICS

(Major)

Paper : MAT0100104–N

(Classical Algebra)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer thc following : 1×8=8

   (a) Define skew-symmetric matrix.

   (b) Write the expansion for e^x.

   (c) Define upper triangular matrix.

   (d) Fill in the blank :

   $\sin h^2 x - \cos h^2 x =$ ______

(e) State the De Moivre's theorem.

(f) Find number of positive and negative real roots of the equation

$x^6 - 3x^2 - 2x - 3 = 0$.

(g) Find number of complex roots of the equation $x^7 - 4x^3 - x + 1 = 0$.

(h) What type of equation $4x^5 - x + 1 = 0$ is ?

2. Answer ***any six*** from the following : 2×6=12

(a) Show that $i^i = e^{-(4n+1)\frac{\pi}{2}}$.

(b) Find all the complex numbers z such that $e^z = 1 + \sqrt{3}i$.

(c) Prove that

$\log(1 + i\tan\alpha) = \log\sec\alpha + i\alpha$.

(d) Prove that

$$\frac{\pi^2}{2.4} - \frac{\pi^4}{2.4.6.8} + \frac{\pi^6}{2.4.6.10.12} - \cdots = 1$$

(e) Find the rank of the matrix

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 6 & 3 & 7 \\ 2 & 1 & 1 \end{bmatrix}.$$

(f) Check whether $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal or not.

(g) Find $Log\, Z$ and $log\, Z$ where $Z = 1$.

(h) If α is a root of the equation $f(x) = 0$, then prove that $(x - \alpha)$ is a factor of the polynomial $f(x) = 0$.

(i) Solve the equation $x^3 + 5x^2 + 2x - 8 = 0$, if one root of the equation is -2.

(j) Form an equation whose roots are reciprocal of the roots of the equation

$$x^4 - 4x^3 + 2x^2 + x + 6 = 0.$$

3. Answer ***any four*** from the following : 5×4=20

(a) Find the relation of the coefficient p, q, r, s that the sum of two roots of the equation is 0.

$$x^4 + px^3 + qx^2 + rx + s = 0.$$

(b) Solve the equation $x^3 + 5x^2 + 7x + 2 = 0$, given that the product of two of the roots is 1.

(c) Show that the following homogenous system of linear equations has infinitely many solutions :

$$x + 2y + 2z = 0$$
$$2x + 5y + 7z = 0$$
$$3x + 6y + 6z = 0$$

(d) Find inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{bmatrix}.$$

(e) Express $Z = -1 - i$ in polar form.

(f) Find cube roots of unity.

(g) Using De Moivre's theorem prove that

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$

(h) If $2\cos\theta = x + \frac{1}{x}$ and θ is real, prove that $2\cos n\theta = x^n + \frac{1}{x^n}$, where n being an integer.

4. Answer ***any two*** from the following : 10×2=20

(a) Prove that :

(i) $\sin\left(i \log \frac{a - ib}{a + ib}\right) = \frac{2ab}{a^2 + b^2}$ 5

(ii) $\tan\left(i\log\frac{a-ib}{a+ib}\right)=\frac{2ab}{a^2-b^2}$ 5

(b) Solve :

$$2x-3y+z=0$$
$$x-3z+2y=0$$
$$4x-y-2z=0$$

(c) If 'x', 'y' be any real numbers, prove that–

(i) $\sinh(x+iy)=\sinh x.\cos y+i\cosh x.\sin y$ 5

(ii) $\cosh(x+iy)=\cosh x.\cos y+i\sinh x.\sin y$ 5

(d) Solve the following cubic equation by Cardon's method :

$$x^3+9x^2+15x-25=0$$

(e) Solve this equation by Euler's method :

$$x^4+8x^3-34x^2-392x-735=0$$