1 (Sem–3/FYUGP) STA41MJ

2025

STATISTICS

(Major)

Paper : STA4300104 MJ

(Survey Sampling and Design of Experiments–I)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :
   1×5=5

   (a) Sampling errors are not present in ____.
   (Fill in the blank)

   (b) When sample size increases, sampling error also increases.
   (State True or False)

Contd.

(c) Replication provides a valid estimate of
_____. *(Fill in the blank)*

(d) The error d.f. in an RBD with 5 blocks
and 4 treatments is ____.
(Fill in the blank)

(e) A complete list of sampling units which
represent the population to be covered
is called the ____. *(Fill in the blank)*

2. Answer *any five* questions from the
following : 2×5=10

(a) Explain the term "sampling error".

(b) How errors can be controlled by Local
Control ?

(c) What is judgement sampling ?

(d) When do we go for stratification in a
sample survey ?

(e) In what situation will you prefer RBD
over CRD ?

(f) Define uniformity trials.

(g) What is mixed sampling ?

(h) What are the different types of
statistical models for experimental
design ?

(i) Define the term treatment in the design
of experiments.

(j) Write advantages and disadvantages of
CRD.

3. Answer *any four* questions from the
following : 5×4=20

(a) What are the advantages of sample
survey over complete enumeration ?

(b) Obtain the variance of the sample mean in case of SRSWOR. What is the variance in case of SRSWR?

(c) What is Pilot survey? In what situation Pilot survey is to be conducted? Give *one* example.

(d) What is a linear model? Write a note on the assumptions made in a linear model in the AOV.

(e) Define linear and circular systematic sampling. Give the condition under which a systematic sampling is more precise than a SRSWOR sampling.

(f) Prove that in stratified random sampling, the sample mean is an unbiased estimate of the population mean. Also find its sampling variance.

(g) Describe the basic principles of Design of Experiments that are used in the construction of CRD and RBD.

(h) Explain the principal steps involved in the planning and execution of sample survey.

4. Answer *any one* question from the following :

$$10 \times 1 = 10$$

(a) (i) An experiment was conducted to test 4 treatments A, B, C and D in 5 randomized blocks. Under the above set-up answer the following :

1. Write the null hypothesis.

2. Construct the AOV table.

3. Draw conclusion.

(ii) The following random sample has been drawn from a population of size 200 :

35, 15, 55, 50, 41, 25, 30, 35, 47, 40.

Estimate the population mean and its standard error.

(b) Considering the linear cost function

$$C = a_0 + \sum_{i=1}^{n} c_i n_i$$

a_0 being the overhead cost and c_i the cost per unit for the i-th stratum, obtain the optimum value of n_i, $(i = 1, 2, \cdots, n)$.

(c) Find the unbiased estimate of the population mean in linear systematic sampling. If ρ is the interclass correlation co-efficient between the units of the same systematic sample, show that

$$Var(\bar{y}_{sys}) = \frac{nk-1}{nk}\{1 + (n-1)\rho\}\frac{S^2}{n}$$

What is the minimum values of ρ ?

(d) (i) Write a note on non-sampling biases

(ii) Give complete analysis of an AOV one-way classified data.

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2025

STATISTICS

(Major)

Paper : STA4300204 MJ

(Statistical Inference–2)

Full Marks : 45

Time : 2 hours

**The figures in the margin indicate
full marks for the questions.**

1. Answer the following questions as directed :
   1×5=5

   (a) The probability of type-I error is called
   _____.   *(Fill in the blank)*

   (b) If *t* is a consistent estimator for θ then
   t^2 is a consistent estimator of θ^2.
   (State True or False)

   (c) Neymann Pearson Lemma provides

       (i) an unbiased test

(ii) a most powerful test

(iii) an uniformly most powerful test

(Choose the correct answer)

(d) Suppose we put forward an interval which we expect to include the true parameter value, then the process is called _____ estimation.

(Fill in the blank)

(e) If α and β represent type-I and type-II error respectively, then the power of the test is _____.

(Fill in the blank)

2. Answer **any five** questions from the following :   2×5=10

(a) Distinguish between estimate and an estimator.

(b) Show that if t is an unbiased estimater for θ, then t^2 is a biased estimator of θ^2.

(c) Show that in a sampling from $N(\mu, \sigma^2)$ population, then sample mean is a consistent estimator of μ.

(d) Define a Most Powerful Test.

(e) Show that if a sufficient estimator exists, it is a function of the MLE.

(f) Define a Critical Region.

(g) Describe Kolmogorov–Smirnov test for goodness of fit.

(h) Write *two* assymptotic properties of likelihood ratio test.

(i) State the invariance property of consistent estimator.

(j) Show that unbiased estimators does not always exist.

3. Answer **any four** questions from the following :   5×4=20

(a) Let $x_1, x_2, \cdots, x_n$ be *i.i.d.* $N(\mu, \sigma^2)$ variates.

Show that, $S^2 = \dfrac{1}{n-1}\sum\limits_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of σ^n.

(b) "An MLE may not always exist". Explain with the help of an example

(c) Let $x_1, x_2, \cdots, x_n$ be *i.i.d.* random variables with common p.d.f. .

$$P(x_i = k) = \frac{1}{N}, \quad \begin{matrix} k = 1, 2, \cdots, N \\ i = 1, 2, \cdots, n \end{matrix}$$

Then show that $T = x_{(n)}$ is sufficient for N.

(d) Let $x_1, x_2, \cdots, x_n$ be *i.i.d.* $b(n, p)$ variates where n and p are both unknown. Obtain their estimators using method of moments.

(e) Find the most powerful test of size α for testing $H_0 : \beta = 1$ against $H_1 : \beta = \beta_1$ based on a sample of size 1 from a population with p.d.f.

$$f(x, \beta) = \beta x^{\beta-1} \quad , 0 < x < 1$$

=0, otherwise.

(f) Let p be the probability that coin will show head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and also the power of the test.

(g) Obtain the minimum variance bound (*MVB*) estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known.

(h) Define sign test. Differentiate between one sample and two sample sign test.

4. Answer **any one** question from the following :

$$10 \times 1 = 10$$

(a) Define consistent estimator. State and prove the sufficient condition for consistency.

(b) (i) Obtain the most general form of the distribution differentiable in θ, for which sample mean is the M.L.E.

(ii) Let $x_1, x_{2,} \ldots x_n$ be a random sample from an exponential distribution with p.d.f.

$$f(x, \theta) = \theta e^{-\theta x} \quad , x \geq 0$$

show that $(n-1)/n\bar{x}$ is an unbiased estimator of θ.

(c) (i) With the help of an example show how Crammer Rao inequality help us to find UMVUE of a parameter.

(ii) Give the concept of distribution free method. Describe Kolmogorov–Smirnov test for two samples, explaining clearly it's purpose, assumptions and hypothesis.

(d) (i) Define a complete statistic.

Let $x_1, x_2, ..., x_n$ be a sample from Bernoulli distribution

$$f(x, \theta) = \theta^x (1-\theta)^{1-x}, \quad x = 0,1$$

$$=0 \qquad , \text{ elsewhere}$$

show that $\sum_{i=1}^{n} x_i$ is a complete

statistic for θ.

(ii) Explain the Likelihood ratio test. Let $x_1, x_2, ..., x_n$ be a random sample from $N(\mu, \sigma^2)$, where σ^2 is known. Develope a Likelihood Ratio test for testing $H_0 : \mu > \mu_0$ against $H_1 : \mu > \mu_1$.

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