2 0 1 6

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks
for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

1. (a) The coordinates of the two points P and
Q are $(3, 4, -6)$ and $(1, -2, 3)$. Find $\overrightarrow{PQ}$.   1

   (b) What is the geometrical interpretation of
the vector product of two vectors?   1

   (c) Find $(\vec{a} \cdot \vec{\nabla})\vec{r}$.   1

   (d) Define scalar field and vector field in a
region R in space.   1

(*Turn Over*)

2. (a) If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angel between them, then show that $\sin \dfrac{\theta}{2} = \dfrac{1}{2} |\hat{a} - \hat{b}|$. 2

(b) Show that two non-zero vectors

$$\vec{A} = A_1 \hat{i} + A_2 \hat{j} + A_3 \hat{k} \text{ and } \vec{B} = B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}$$

are parallel if and only if

$$\dfrac{A_1}{B_1} = \dfrac{A_2}{B_2} = \dfrac{A_3}{B_3}$$ 2

(c) If $\dfrac{d}{dt}\vec{u} = \vec{w} \times \vec{u}$ and $\dfrac{d}{dt}\vec{v} = \vec{w} \times \vec{v}$, then

show that $\dfrac{d}{dt}(\vec{u} \times \vec{v}) = \vec{w} \times (\vec{u} \times \vec{v})$. 2

(d) If $\vec{A}$ and $\vec{B}$ are irrotational, then prove that $\vec{A} \times \vec{B}$ is solenoidal. 2

3. Answer any *two* questions : 4×2=8

(a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, then express $\vec{b} \times \vec{c}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(b) Prove that

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

(c) Find $\vec{\nabla} \cdot \vec{F}$ and $\vec{\nabla} \times \vec{F}$, where

$$\vec{F} = \vec{\nabla}(x^3 + y^3 + z^3 - 3xyz)$$

GROUP—B

(Mechanics)

(Marks : 40)

4. (a) Write two factors on which Coriolis force depends. 1

(b) Give one example of conservative force, which is velocity-dependent. 1

(c) Name the physical quantity which is the cause of rotational motion. Is it a scalar or vector quantity? 1

(d) There is a stick of half wood and half steel. The mass of the wooden portion is less than the steel portion. It is pivoted at the wooden end and force is applied at right angle to its length at the steel end. Next it is pivoted at the steel end and the same force is applied at the wooden end. In which case is the angular momentum more and why? 1

(e) A thin rod of length L is bent to form a circle and has a mass M. What will be the gravitational potential at the centre? 1

5. (a) A non-inertial frame is rotating with angular speed 10 rad/s. A body of mass 20 kg is at a distance 20 cm from the axis of rotation. Find the magnitude of the centrifugal force on the body. 2

(b) Show that, $\vec{F} = (2xy + z^2)\hat{i} + x^2\hat{j} + 2xz\hat{k}$ is a conservative force.   2

(c) An observer in Lab. frame finds the velocities of the two particles A and B of masses m_1 and m_2 as v_1 and v_2. Calculate the velocity of each particle and total momentum of the system in centre of mass frame.   2

(d) State the two theorems of moment of inertia.   2

(e) Gravitational potential is always negative in sign. What conclusion can you draw from this statement? Write the relation between gravitational intensity and gravitational potential.   2

6. Answer any *three* questions :   5×3=15

(a) Find the expression of acceleration in non-inertial frame of reference.

(b) Find the centre of mass of a homogenous semicircular disc.

(c) Show that for compound pendulum, the points of suspension and oscillation are interchangeable.

(d) Derive the expression of gravitational potential at a point outside a spherical shell.

   (Continued)

7. Answer any *one* question :   10

(a) (i) What are laboratory and centre of mass frames of reference?   2

     (ii) Obtain a relation of scattering angles in these two frames of reference.   8

(b) (i) Calculate the moment of inertia of spherical shell about its diameter.   7

     (ii) Two bodies of mass 10 kg and 2 kg are moving with velocities $(2\hat{i} - 7\hat{j} + 3\hat{k})$ and $(-10\hat{i} + 5\hat{j} - 3\hat{k})$. Find the velocities of their centres of mass.   3

(c) (i) Derive an expression of deflection of a freely falling body by the Coriolis force due to rotation of earth.   5

     (ii) Two bodies of masses M_1 and M_2 are placed at d distance apart. Show that at the position where the gravitational field due to them is zero, the potential is given by

$$V = -\frac{G}{d}[M_1 + M_2 + 2\sqrt{M_1 M_2}]$$ 5

★ ★ ★

2 0 1 6

PHYSICS

(Major)

Paper : 1.2

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—I

(Waves and Oscillations)

(*Marks* : 40)

Answer Question No. 1 and *any* **five** from the rest

1. (a) Define a stationary wave.   1

   (b) If two waves are represented by
       $x_1 = a \sin(\omega t + \pi/5)$ and $x_2 = a \cos(\omega t - \pi/3)$,
       then what is the phase difference
       between the two waves?   1

   (c) What is the ratio between the intensities
       of the first and fifth harmonics
       produced in a spring plucked at the
       midpoint?   1

A7/**47**   (*Turn Over*)

(d) Give an example of application of Fourier series analysis. 1

(e) What is reverberation in sound wave? 1

2. Show that the direction of the acceleration and force are opposite to the direction of motion of the vibrating particle in simple harmonic motion. Calculate the total energy of a vibrating particles in simple harmonic motion. 3+4

3. (a) Two tuning forks A and B are of nearly equal frequencies. Frequency of A is 256. When the two tuning forks are used to obtain Lissajous figures, the complete cycle of changes takes place in 10 seconds. When the tuning fork B is loaded with a little wax, the time taken is 20 seconds. Calculate the frequency of B before loading. 4

(b) Find whether the discharge of a condenser through the following inductive circuit is oscillatory :

$C = 0 \cdot 1\,\mu F$, $L = 10$ mH, $R = 200\,\Omega$

If it is oscillatory, find the frequency of oscillation. 3

4. (a) State the differences between transverse wave and longitudinal wave. 2

(b) Determine the distribution of velocity and change in pressure in a plane progressive wave. Plot the displacement, velocity and the change in pressure to compare them. 3+2

5. (a) Obtain the positions of the nodes and antinodes of a standing wave formed in a closed-end organ pipe. 5

(b) Give an example each of free, damped and forced oscillations. 2

6. (a) Calculate the velocity of sound in a gas in which two waves of wavelengths 1 meter and 1·01 metre produce 10 beats in 3 seconds. 3

(b) What is the role of reverberation in constructing a good auditorium? Calculate the reverberation time of an auditorium of volume 5500 m³ and absorbing surface area of 750 m². [Given the average absorption coefficient is 0·5.] 2+2

7. Analyze, with the help of Fourier theorem, a periodic wave given by $f(t) = 1$ for $0 \le t \le T/2$ and $f(t) = 0$ for $T/2 \le t \le T$. Also plot the first three terms of the Fourier series. 5+2

8. *(a)* Deduce the expression for the energy of a string vibrating transversely. **5**

(b) Calculate the change in intensity level when the intensity of sound increases 200 times of its original intensity. **2**

9. *(a)* Find the expression of velocity of a longitudinal wave in a thin solid bar. **5**

(b) Distinguish between phase velocity and group velocity. **2**

SECTION—II

(Ray Optics)

(Marks : 20)

Answer *any* **four** questions

10. Using Fermat's principle, derive the formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ for a spherical mirror. **5**

11. Using matrix method, find the matrix for combination of two lenses of focal lengths f_1 and f_2 and separated by a distance d in air. **5**

12. Obtain the lens formula for a thick lens of thickness t. **5**

(*Continued*)

13. Derive the relation between lateral magnification and longitudinal magnification of a lens system. **5**

14. Show that spherical refracting surface is aplantic with respect to certain position of the object. **5**

15. *(a)* What do you mean by aberration in an optical system? **1**

(b) Distinguish between coma and astigmatism. **2**

(c) What is achromatic doublet? **2**

★ ★ ★