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1 (Sem-4) STA 5

2025

STATISTICS

Paper : STA0400504

(Linear Algebra and System of Equations)

Full Marks : 60

Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

1. Answer the following questions as directed:

1×8=8

- (a) State one property of Determinants.
- (b) The rank of the transpose of a matrix.
is the same as that of the _____
matrix. (Fill in the blank)
- (c) Which of the following is standard form
of Characteristic equation ?
 - (i) $(A - \lambda I) = 0$
 - (ii) $(A - \lambda I)X = 0$
 - (iii) $|A - \lambda I| = 0$
 - (iv) $|A - \lambda I| = I$

(Choose the correct option)

(d) The product of two determinants of same order is itself a determinant of that order. (True or False)

(e) If A is non-singular matrix, then

(i) $|A^{-1}| = 0$

(ii) $|A^{-1}| = |A|^{-1}$

(iii) $|A^{-1}| = I$

(iv) $|A| = 0$

(Choose the correct option)

(f) The intersection of any two subspace of a vector space is also a subspace. (True or False)

(g) The matrix $\begin{bmatrix} 1 & 4 & 7 \\ 4 & 2 & 3 \\ 7 & -3 & 1 \end{bmatrix}$ is

(i) Symmetric matrix

(ii) Skew Symmetric matrix

(iii) Hermitian matrix

(iv) None of the above

(Choose the correct option)

(h) A subset S of a vector space is said to be a basis if S consists of linearly _____ vector. (Fill in the blank)

2. Answer **any six** questions from the following: $2 \times 6 = 12$

(a) Show that value of a determinant of a skew symmetric matrix of odd order is always zero.

(b) Show that every super set of a linearly dependent set of vectors is linearly dependent.

(c) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$$

(d) Write down the system of m simultaneous non-homogeneous linear equations in n variables. Also define Augmented Matrix.

(e) Write down the quadratic forms corresponding to the matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}.$$

- (f) Show that the two matrices A , $C^{-1}AC$ have the same characteristic roots.
- (g) Show that the three vectors $(1, 2, 0)$, $(0, 3, 1)$ and $(-1, 0, 1)$ of R^3 are linearly independent.
- (h) Find the inverse of the following matrix using Elementary transformations

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}.$$

- (i) State the condition under which a system of non-homogeneous equations will have —

- (a) No Solution
- (b) Unique Solution

- (j) Find the characteristic roots for the matrix A , where

$$A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}.$$

3. Answer **any four** questions from the following:

5×4=20

- (a) State the properties of vector space.

- (b) Find the ranks of A , B , $A+B$, AB and BA where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}.$$

- (c) Show that the vectors $(1, 2, 1)$, $(2, 1, 0)$ and $(1, -1, 2)$ forms a basis of R^3 .
- (d) Determine the characteristic roots and corresponding characteristic vectors of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

- (e) Write down the procedure of Jacobi's iterative method to solve simultaneous linear algebraic equations.
- (f) Prove that the following systems of equations are consistent

$$x - y + z = 2$$

$$3x - y + 2z = 6$$

$$3x + y + z = -18$$

(g) Solve the following equation for x

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0.$$

(h) Solve the system of non-homogeneous linear equations

$$x - 2y + 3z = 5$$

$$4x + 3y + 4z = 7$$

$$x + y - z = -4; \text{ using Cramer rule.}$$

4. Answer **any two** questions from the following: $10 \times 2 = 20$

(a) State and prove the Cayley-Hamilton Theorem.

(b) If x, y, z are all different and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$$

then prove that $xyz = -1$.

(c) Let Δ be the determinant of order n and D is the adjoint of Δ . Then show that $D = \Delta^{n-1}$.

(d) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix},$$

and verify that it is satisfied by A and hence obtain A^{-1} .

(e) Prove that the rank of a product of two matrices can not exceed the rank of either of the matrices.
