2018

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 20)

3. *(a)* Find the Cartesian component of a vector $\vec{C}$ which is perpendicular to the vector $\vec{A}$ and vector $\vec{B}$, where

$$\vec{A} = 2\hat{i} - \hat{j} + \hat{k} \quad \text{and} \quad \vec{B} = 3\hat{i} + 4\hat{j} - \hat{k}$$ 1

(b) Define vector field in a region of space. Give an example of vector field. 1

2. *(a)* Give the vector diagram representation of $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{B}\times\vec{A}=\vec{D}$. Name a physical vector quantity which is the product of two vectors. 2

(b) What is the physical significance of divergence of a vector? 2

(c) Find the projection of vector $\vec{A}$ on vector $\vec{B}$, where $\vec{A}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{B}=\hat{i}-3\hat{j}+4\hat{k}$. 2

(d) A particle with position vector $\vec{r}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$ moves with a constant angular velocity ω. The linear velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$. Show that $\vec{r}\times\vec{v}$ is a constant vector. 2

3. Answer any *two* questions : 5×2=10

(a) If $\bar{V}=\bar{a}\cos\omega t+\bar{b}\sin\omega t$, find that

$$\bar{V}\times\frac{d\bar{V}}{dt}=\omega(\vec{a}\times\vec{b})$$

Here $\vec{a}$ and $\vec{b}$ are two constant non-linear vectors and ω is constant scalar.

(b) If $r=(x^2+y^2+z^2)^{1/2}$, show that

$$\nabla^2\left(\frac{1}{r}\right)=0$$

(c) Show that gradient of any scalar field $\phi(r)$ is irrotational and the curl of any vector field $\vec{V}(r)$ is solenoidal.

GROUP—B

(Mechanics)

(*Marks* : 40)

4. *(a)* What is fictitious force? Give an example of it. 1

(b) Is the centre of mass frame of reference an inertial frame? Explain. 1

(c) A particle is moving horizontally at the equator. What is the value of Coriolis force acting on it in local coordinate system? 1

(d) What is the difference between laboratory frame of reference and centre of mass frame of reference? 1

(e) When is a force field said to be conservative? Give an example of conservative force. 1

(f) Can we have equipotential surfaces of the gravitational field of a point mass? What is the value of work done if a mass moves on an equipotential surface? 1

5. (a) Two particles of mass 2 kg each are moving with velocity $(2\hat{i}+4\hat{j})$ m/s and $(5\hat{i}+6\hat{j})$ m/s respectively. Find the kinetic energy of the system relative to centre of mass. 2

(b) Show that force field given by $\vec{F} = x^2yz\hat{i} - xyz^2\hat{k}$ is non-conservative. 2

6. Answer any *two* questions : 5×2=10

(a) Show that whenever a body is acted upon by a number of forces such that the resultant is not zero, then the work done by the resultant force is equal to the change in the kinetic energy of the body.

(b) Calculate the moment of inertia of a thin hollow sphere about its diameter.

(c) Find the centre of mass of a uniform solid hemisphere of radius a.

7. Answer any *two* questions : 10×2=20

(a) (i) Distinguish between inertial mass and gravitational mass.

(ii) Obtain an expression for the gravitational potential and field due to a thin uniform spherical shell at an external point.

(iii) The radius of the earth is $6{\cdot}637 \times 10^6$ m and its mean density is $5{\cdot}57 \times 10^3$ kg/m^3. Calculate earth surface potential. Given $G = 6{\cdot}66 \times 10^{11}$ Nm^2 kg^{-2}. 2+5+3=10

(b) (i) What is the effect of Coriolis force on a particle falling freely under the action of gravity?

(ii) Show that the angular accelerations of a particle in a fixed system and a rotating system are same. 5+5=10

(c) (i) Give a schematic diagram of elastic collision of two particles in centre of mass frame and laboratory frame.

(ii) Obtain a relation of scattering angles in these two frames of reference. 2+8=10

(d) (i) Prove that a conservative force can be expressed as negative gradient of potential.

(ii) Two particles of masses m_1 and m_2 separated by infinite distance apart, attract each other according to the law of gravitation. Considering the particles to be initially at rest, show that their velocity of approach

$$v = \sqrt{\frac{2G(m_1 + m_2)}{a}}$$

where a is final separation of the two masses.

(iii) Find the force field associated with the potential energy $V = Ae^{\alpha(x+y+z)}$, where A and α are constants. 4+4+2=10

★ ★ ★

2018

PHYSICS

(Major)

Paper : 1.2

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

SECTION—I

(Waves and Oscillations)

(*Marks* : 40)

1. *(a)* What is the phase difference between the displacement and acceleration of a particle executing SHM? 1

(b) A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Write the wave equation of the unknown wave. 1

(c) What is reverberation of sound? 1

(d) The function $f(x) = x^2$ is defined within the interval $-\pi \le x \le \pi$ and outside it is periodic. State whether the function is even or odd within $-\pi \le x \le \pi$. 1

(Turn Over)

2. *(a)* What is sharpness of resonance? Explain the effect of damping on the sharpness of resonance. 2

(b) The phase velocity V depends on the wavelength λ according to relation $V = A\sqrt{\lambda}$, where A is constant. Show that group velocity is half of the phase velocity. 2

(c) If the displacement x and velocity V of a particle executing simple harmonic motion are related through the expression $4V^2 = 25 - x^2$, then calculate its time period. 2

3. Answer any *two* questions : 5×2=10

(a) Show that in case of damped oscillation the loss of energy is equal to the rate of work done against the resistive force.

(b) A particle is simultaneously subjected to two simple harmonic motions moving in the same direction, each of same frequency but of different amplitude. If phase difference between them is $\pi/4$, find the amplitude of the resultant motion and the phase relation to one of the components.

(c) Derive the expression for the velocity of transverse wave propagating in a stretched string under tension.

(Continued)

Answer any *two* questions :

4. Find the Fourier series for a function

$$f(x) = 0, \text{ for } -\pi < x < 0$$
$$= h, \text{ for } 0 < x < \pi$$

What are the conditions for a function which can be expanded by Fourier series? 7+3=10

5. What are beats? Give an analytical description of the phenomenon of beats. Show that the beat frequency is equal to the difference of frequencies of the component oscillations. 2+4+4=10

6. *(a)* Show that intensity of sound wave at a point is given by

$$I = \frac{P_{rms}^2}{\rho V}$$

where P_{rms} is root mean square velocity of excess pressure, ρ is the density of the gaseous medium and V is the velocity of sound. 6

(b) If intensity leyel of a sound is increased by 1 dB, then calculate the percentage increase of intensity of the sound. 4

(Turn Over)

7. *(a)* A transverse wave is represented by

$$y = y_0 \sin \frac{2\pi}{\lambda}(vt - x)$$

Find the value of λ for which the maximum particle velocity becomes equal to twice the wave velocity. 4

(b) For a particle executing SHM, show that its average kinetic energy is equal to half of its total energy. 6

SECTION—II

(Ray Optics)

(*Marks* : 20)

Answer *any* **four** questions

8. State Fermat's principle for stationary path with the mathematical relation of optical path variation. Establish the Fermat's principle for refraction at curved surface. 2+3=5

9. What do you mean by translation matrix? Find out an expression of translation matrix which transforms a ray $\begin{bmatrix} \lambda_1 \\ x_1 \end{bmatrix}$ into the ray $\begin{bmatrix} \lambda_2 \\ x_2 \end{bmatrix}$ during translation through a distance d in a homogenous medium. 1+4=5

(*Continued*)

10. A concave lens is placed at a distance of 25 cm in front of a concave mirror of focal length 20 cm. It is found that a pin placed at a distance of 45 cm in front of the lens coincide with its own inverted image formed by the combination. Using refraction matrix, find the focal length of the lens. 5

11. What is spherical aberration in a lens? What is circle of least confusion in this aberration? Find out the condition for minimisation of spherical aberration by using two lenses separated by finite distance. 1+1+3=5

12. Write a short note on any *one* of the following : 5

(a) Chromatic aberration and its elimination

(b) High power oil immersion objective

★ ★ ★