2018

MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×10=10

   (a) What is the condition that union of two subgroups of a group is again a subgroup of the group?

   (b) What is the order of element

   $$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 4 & 2 & 5 & 1 & 7 & 8 & 9 \end{pmatrix}$$

   of the permutation group P_9?

(c) Is every subgroup of an Abelian group is normal?

(d) If I_n be a unit matrix of order n, then what is the matrix adj I_n?

(e) What is the normal form of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$?

(f) If the non-singular matrix A is symmetric, then

(i) A is Hermitian

(ii) A is skew-Hermitian

(iii) A^{-1} is symmetric

(iv) A^{-1} is skew-symmetric

(Choose the correct answer)

(g) What is the rank of a non-singular matrix of order 3×3?

(h) Express the complex number $-1+i$ in its polar form.

(i) What is the relation between circular and hyperbolic functions of sine?

(j) What is the value of $\log_e i$?

2. Answer the following questions : 2×5=10

(a) If a is a generator of a cyclic group G, then show that a^{-1} is also a generator of G.

(b) If A is a symmetric matrix, then prove that adj A is also symmetric.

(c) With an example, show that a matrix which is skew-symmetric is not skew-Hermitian.

(d) If A and B be two equivalent matrices, then show that rank A = rank B.

(e) If $x + \frac{1}{x} = 2\cos\theta$, then show that

$$x^n + \frac{1}{x^n} = 2\cos n\theta$$

3. Answer the following questions : 5×2=10

(a) If H is a subgroup of a group G and N is a normal subgroup of G, then show that $H \cap N$ is a normal subgroup of H.

(b) Prove that n, nth roots of unity forms a series in GP.

Or

Show that

$$1-\frac{2}{\underline{|3}}+\frac{3}{\underline{|5}}-\frac{4}{\underline{|7}}+\cdots\infty=\frac{1}{\sqrt{2}}\sin\left(\frac{\pi}{4}+1\right)$$

4. Answer any *two* questions : 5×2=10

(a) If α, β, γ are the roots of the equation $x^3+qx+r=0$, then form an equation whose roots be $(\alpha-\beta)^2, (\beta-\gamma)^2, (\gamma-\alpha)^2$.

(b) Solve the equation by Cardon's method

$$x^3+6x^2+9x+4=0$$

(c) If $A, B, \cdots, L; a, b, \cdots, l; m \in R$, then prove that

$$\frac{A^2}{x-a}+\frac{B^2}{x-b}+\cdots+\frac{L^2}{x-l}=x+m$$

has all its roots real.

5. Answer either *(a)* or *(b)* : 10

(a) Prove that a mapping $f: X \to Y$ is one-one onto iff there exists a mapping $g: Y \to X$ such that $g \circ f$ and $f \circ g$ are identity maps on X and Y, respectively.

(b) Show that an equivalence relation R in a non-empty set S determines a partition of S and conversely, a partition of S defines an equivalence relation in S.

6. Answer either *(a)* or *(b)* :

(a) If H and K be two subgroups of a group G, then prove that HK is a subgroup of G iff $HK = KH$. $[HK = \{hk : h \in H, k \in K\}]$ 10

(b) Prove that order of each subgroup of a finite group is a divisor of the order of the group. Hence prove that if G is a finite group of order n and $a \in G$, then $a^n = e$. 6+4=10

7. Answer either *(a)* or *(b)* :

(a) If $\tan(\alpha + i\beta) = x + iy$, then find x and y. Hence show that $x^2 + y^2 + 2x\cot 2\alpha = 1$. 10

(b) *(i)* If $x < \sqrt{2} - 1$, then prove that

$$2\left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \dots\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^5 - \cdots$$

(ii) Show that

$$\frac{\pi}{12} = \left(1 - \frac{1}{3^{1/2}}\right) - \frac{1}{3}\left(1 - \frac{1}{3^{3/2}}\right) + \frac{1}{5}\left(1 - \frac{1}{3^{5/2}}\right) - \dots \infty$$

5+5=10

8. Answer either *(a)* or *(b)* :

(a) If A and B are two square matrices of the same order, then prove that

$$\text{adj}\,(AB) = (\text{adj}\,B)\cdot(\text{adj}\,A)$$

Verify it for the matrices

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}, \; B = \begin{bmatrix} 6 & 1 \\ -1 & 3 \end{bmatrix}$$ 6+4=10

(Continued)

(b) What is normal form of matrix of a rank r? Find the rank of the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

by reducing it to normal form. 2+8=10

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MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write nth derivative of $\log(ax+b)$.

(b) If $z = f(y/x)$, what is the value of

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$$

(c) Write the formula for radius of curvature of a Cartesian curve.

(d) Two curves $y = f(x)$ and $y' = g(x)$ intersect at the point (x_1, y_1). Find the condition that they cut orthogonally.

(e) Define double point of a curve.

(f) What is the volume of the solid generated due to the revolution of the circle $x^2 + y^2 = 4$ about X-axis?

(g) Write subnormal to the curve $y^2 = 4ax$ at any point (x, y).

(h) Write the value of $\int_0^{\pi/2} \cos^7 x \, dx$.

(i) Write the value of

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$$

(j) Write the maximum number of asymptotes of algebraic curve of nth degree.

2. Solve the following questions : 2×5=10

(a) If $y = e^{ax} \sin bx$, show that

$$y_2 - 2ay_1 + (a^2 + b^2)y = 0$$

(b) If $x = r\cos\theta$, $y = r\sin\theta$, prove that

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0, \; x \neq 0, \; y \neq 0$$

(c) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(d) Find the value of $\int_0^{\pi} x \cos^4 x \, dx$.

(e) Find the equation of tangent to the curve $y = be^{-x/a}$ at the point, where it crosses the axis of y.

3. Answer the following questions : 5×2=10

(a) If $y = \sin^{-1} x$, find $(y_n)_0$ where n is odd.

(b) Obtain a reduction formula for $\int \sec^n x \, dx$.

4. Answer either part (a) or part (b) : 10

(a) (i) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

(ii) Find the points of inflexion of the curve $y(a^2 + x^2) = x^3$. 5+5=10

(b) (i) If $u = f(x, y)$, where $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(ii) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with x-axis, show that its equation is

$$y\cos\phi - x\sin\phi = a\cos 2\phi.$$

5+5=10

(Continued)

5. Answer the following questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \log \sin x \, dx$$

(b) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6. Answer part (a) or part (b) : 10

(a) (i) Obtaining nth derivative of x^{2n}, prove that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \cdot 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \cdot 2^2 \cdot 3^2} + \cdots = \frac{\lfloor 2n}{(\lfloor n)^2}$$

(ii) Show that the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $\dfrac{3}{8}\pi a^2$.

5+5=10

(b) (i) Find the asymptotes of the curve

$$x^3 + y^3 - 3axy = 0$$

(ii) Trace the curve $r = a(1 + \cos\theta)$.

5+5=10

(Turn Over)

7. Answer any *two* questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \frac{dx}{5+3\cos x}$$

(b) If $I_n = \int (a^2 + x^2)^{n/2}\, dx$, show that

$$I_n = \frac{x(a^2 + x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$

(c) Evaluate :

$$\int \frac{dx}{(x^2 - 2x + 1)\sqrt{x^2 - 2x + 3}}$$

8. Answer the following questions : 5×2=10

(a) Integrate :

$$\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$$

Or

$$\int \frac{dx}{(1-x)\sqrt{1-x^2}}$$

(Continued)

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

Or

Find the surface area of the solid generated by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line.

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