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3 (Sem-3/CBCS) MAT SE 1/SE 2

2024

MATHEMATICS

(Skill Enhancement Course)

Answer the Questions from any one Option.

OPTION - A

Paper : MAT-SE-3014

**(Computer Algebra Systems
and Related Software)**

OPTION - B

Paper : MAT-SE-3024

(Combinatorics and Graph Theory)

Full Marks : 50

Time : Two hours

***The figures in the margin indicate
full marks for the questions.***



Contd.

OPTION - A

Paper : MAT-SE-3014

(Computer Algebra Systems and Related Software)

1. Answer the following questions : $1 \times 4 = 4$

- (a) What is a computer algebra system?
- (b) Write the Mathematica and Maple commands to calculate π up to 100 decimals.
- (c) Write *any two* computer algebra systems' commands to calculate $\sum_{i=1}^5 i^2$.
- (d) Write *any one* computer algebra system expression to calculate the dot product of two vectors (a_1, a_2, a_3, a_4) , (b_1, b_2, b_3, b_4) .

2. Answer the following questions : $2 \times 3 = 6$

- (a) What are the roles of the keyboard keys ENTER and SHIFT + ENTER in Mathematica and Maple softwares?
- (b) What is the utility of 'Clear' command in Mathematica? Write a command in Maxima which works similar to 'Clear' command of Mathematica.

(c) In Mathematica notebook, write the procedure to delete all outputs at a time. How can you evaluate all inputs at a time?

3. Answer *any two* parts : $5 \times 2 = 10$

(a) Write the output of the following built-in functions' values in Mathematica :

- (i) Binomial[7, 2]
- (ii) FactorInteger[625]
- (iii) Prime[5]
- (iv) D[sin[x], x]
- (v) Power[Power[3, 2], 2]

(b) Suppose $f(x) = x^3 + \sin x$. In any two computer algebra systems, define the function suitably and write the input statements $f(\pi)$. Write the commands in those systems to calculate the differentiation of the said function.

(c) Explain with examples, two different ways to define a piecewise function in Mathematica.

(d) Write a program in Maxima using user-defined function to find the square root of 81 with the help of Newton's method.

4. Answer **any three** parts : $10 \times 3 = 30$

(a) (i) Let $A = (a_{ij})_{3 \times 3}$ be a matrix. Write the commands in Mathematica to extract the diagonal elements from the matrix and obtain the adjoint of A . Hence write the commands to find inverse of A , without using the Mathematica command 'Inverse[A]'. 5

(ii) Write the Mathematica program to solve the system $AX = 0$, where, A is the 3×3 coefficient matrix, X is the 3×1 matrix containing the variables x, y, z . 0 is the 3×1 zero matrix. 5

(b) Let $v_1 = (10, 4, 5), v_2 = (4, 4, 7), v_3 = (8, 1, 0), b = (1, 2, 3)$ be four vectors in R^3 . Write a Mathematica program to check whether b is a linear combination of the vectors v_1, v_2, v_3 or not. Write the possible outputs of each command from the program. Also check whether v_1, v_2, v_3 are linearly independent or not. Discuss your conclusion from the program output.

(c) Gram-Schmidt process of orthogonalization for the vectors

$v_1, v_2, \dots, v_n \in R^n$ is

$$u_1 = v_1, u_k = v_k - \sum_{i=1}^{k-1} \frac{\langle v_k, u_i \rangle u_i}{\langle u_i, u_i \rangle}, k = 2, 3, \dots, n$$

$\langle a, b \rangle$ represents dot product of a and b . Further Gram-Schmidt orthonormal

set is $\left\{ \frac{u_1}{|u_1|}, \frac{u_2}{|u_2|}, \frac{u_n}{|u_n|}, \dots, \frac{u_1}{|u_1|} \right\}$.

Let $v_1 = (10, 4, 5), v_2 = (4, 4, 7),$

$v_3 = (8, 1, 0)$ be three vectors. Write the Mathematica command to obtain the Gram-Schmidt orthonormal set. Without using that command, write suitable Mathematica expressions to obtain the Gram-Schmidt orthonormal set from the vectors v_1, v_2, v_3 . Write the output of each step. How can you verify, in Mathematica, whether the set is orthonormal or not?

(d) Write a short note on Mapple as calculator. Give examples.

(e) Write a short note on built-in functions and user-defined functions in Maxima. Give examples.

(f) Write the commands in Mathematica, Maple and Maxima for the following problems :

- (i) Plotting $y = \sin x$, $0 \leq x \leq 2\pi$;
- (ii) Showing a graph of three lines,
 $y = 4x + 1$, $y = -x + 4$,
 and $y = 9x - 8$, for $0 \leq x \leq 2$;
- (iii) Showing a graph of the surface
 $z = e^{-x^2+y^2}$, $-2 \leq x, y \leq 2$.

OPTION - B

Paper : MAT-SE-3024

(Combinatorics and Graph Theory)

1. Answer the following questions : $1 \times 4 = 4$

তলৰ প্ৰশ্নৰ উত্তৰ কৰা :

(a) Define a graph.

এটা লেখৰ সংজ্ঞা দিয়া।

(b) Write the sum rule in counting.

গণনাৰ যোগৰ সূত্ৰটো লিখা।

(c) You have three different shirts and four different pants. In how many different ways can you dress with pant-shirt?

তোমাৰ তিনিটা ভিন্ন কামিজ আৰু চাৰিটা ভিন্ন পেণ্ট আছে। তুমি কিমান ভিন্ন ধৰণে কামিজ-পেণ্ট পোচাক পিন্ধিব পাৰিবা?

(d) Write True **or** False :

সঁচা নে মিছা লিখা :

“In a graph there are odd number of vertices of odd degree.”

“এটা লেখত অযুগ্ম ঘাত বিশিষ্ট অযুগ্ম সংখ্যক শীৰ্ষবিন্দু থাকে।”

2. Answer the following questions : $2 \times 3 = 6$

তলৰ প্রশ্নৰ উত্তৰ কৰা :

- (a) A computer password contains an alphabet followed by three digits. How many passwords be created, if (i) the digits are not repeated, and (ii) the digits can be repeated?

এটা কম্পিউটাৰৰ পাচৱাৰ্ডত এটা ইংৰাজী বৰ্ণ আৰু তিনিটা অংক আছে। মুঠতে কিমানটা পাচৱাৰ্ড সৃষ্টি হ'ব (i) যদি অংকৰ পুনৰাবৃত্তি নহয়, আৰু (ii) যদি অংকৰ পুনৰাবৃত্তি হয়?

- (b) Draw a diagram of the complete graph K_5 and a diagram of the bipartite graph $K_{3,5}$.

সম্পূৰ্ণ লেখ K_5 আৰু বিপাৰ্টিট লেখ $K_{3,5}$ -ৰ এটাকৈ চিত্ৰ আঁকা।

- (c) Find the number of committees of five formed by 6 men and 5 women such that there are at least two women present in the committee.

6 জন পুৰুষ আৰু 5 জনী মহিলাৰ পৰা কিমানটা 5 জনীয়া কমিটি গঠন কৰিব পাৰি যদি তাত অন্ততঃ দুজনী মহিলা থাকে?

3. Answer **any two** of the following :

$5 \times 2 = 10$

তলৰ যিকোনো দুটা প্রশ্নৰ উত্তৰ কৰা :

- (a) Does there exist a simple graph with five vertices having degrees 2, 2, 4, 4, 4? Justify your answer.

2, 2, 4, 4, 4 ঘাতবিশিষ্ট পাঁচটা শীৰ্ষবিন্দুৰ এটা সৰল লেখ স্থিত হ'বনে? তোমাৰ উত্তৰৰ যথার্থতা ব্যাখ্যা কৰা।

- (b) Which regular polygon has the same number of diagonals and sides?

কোনটো সুৰম বহুভুজৰ সমসংখ্যক কর্ণ আৰু বাহু থাকে?

- (c) Let G be a connected graph with at least two vertices. If the number of edges in G is less than the number of vertices, then prove that G has a vertex of degree one.

অন্ততঃ দুটা শীৰ্ষবিন্দু যুক্ত G এটা সংযুক্ত লেখ। যদি G -ৰ বাহুৰ সংখ্যা শীৰ্ষবিন্দুৰ সংখ্যাতকৈ কম হয়, তেন্তে প্রমাণ কৰা যে G -ৰ একঘাতৰ এটা শীৰ্ষবিন্দু থাকিব।

- (d) How many positive integers not exceeding 600 are divisible by 5 or 13 ?
600 তকৈ সৰু 5 বা 13 ৰে বিভাজ্য নোহোৱা কিমানটা
ধনাত্মক অখণ্ড সংখ্যা আছে?

4. Answer **any three** of the following :

$$10 \times 3 = 30$$

তলৰ যিকোনো তিনিটা প্ৰশ্নৰ উত্তৰ কৰা :

- (a) Prove that a simple graph with n vertices and m components can have at most $(n-m)(n-m+1)/2$ edges.

প্ৰমাণ কৰা যে n টা শীৰ্ষবিন্দু আৰু m টা শাখা থকা
এটা সৰল লেখৰ সৰ্বোচ্চ বাহুৰ সংখ্যা

$$(n-m)(n-m+1)/2.$$

- (b) Find the number of primes less than 144.

144 তকৈ সৰু মৌলিক সংখ্যাৰ মুঠ সংখ্যা নিৰ্ণয় কৰা।

- (c) Prove that there is always a Hamiltonian path in a directed complete graph.

প্ৰমাণ কৰা যে এটা নিৰ্দেশিত সম্পূৰ্ণ লেখত সদায় এটা
হেমিল্টন পথ থাকিব।

- (d) If 24 candidates appear in a competitive examination, then show that there exist at least two among them whose Roll number differ by a multiple of 19.

যদি এটা পৰীক্ষাত 24 গৰাকী প্ৰাৰ্থী অবতীৰ্ণ হয়,
দেখুওৱা যে তাৰ ভিতৰত অন্ততঃ দুজন প্ৰাৰ্থী থাকিব
যাৰ ৰোল নম্বৰৰ পাৰ্থক্য 19ৰ এটা গুণিতক হয়।

- (e) Prove that a connected graph G is an Eulerian graph if and only if all vertices of G are of even degree.

প্ৰমাণ কৰা যে এটা সৰল লেখ G এটা অয়লাৰ লেখ
হব যদি আৰু যদিহে G -ৰ সকলোবোৰ শীৰ্ষবিন্দুৰ ঘাত
যুগ্ম হব।

- (f) Give definition with an example of each of the following :

এটাকৈ উদাহৰণেৰে সৈতে তলত দিয়াবোৰৰ সংজ্ঞা
দিয়া :

- (i) Pigeonhole principle

পিজনহল সূত্ৰ

- (ii) Simple path

সৰল পথ

(iii) Isolated graph

বিযুক্ত লেখ

(iv) Eulerian graph

অয়লাৰ লেখ

(v) Cycle

চক্ৰ

