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MATHEMATICS

(Major)

Paper : 6.1

(Hydrostatics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Define surface of equal pressure.

(b) State Boyle's law.

(c) Define absolute zero of temperature.

(d) Define centre of pressure of a plane area immersed in a fluid.

(e) What is internal energy?

(f) Define the term convective equilibrium for a gas.

(g) Define surface of floatation.

2. Answer the following questions : 2×4=8

(a) Obtain the differential equations of the lines of force at any point (x, y, z).

(b) Prove that the pressure at any point varies as the depth of the point from the surface when there is no atmospheric pressure.

(c) If ρ_0 and ρ be the densities of a gas at 0° and $t°$ centigrade respectively, then establish the relation $\rho_0 = \rho(1+\alpha t)$ where

$$\alpha = \frac{1}{273}$$

(d) Show that the positions of equilibrium of a body floating in a homogeneous liquid are determined by drawing normals from G, the centre of mass of the body, to the surface of buoyancy.

3. Answer any *three* parts : 5×3=15

(a) If a mass of fluid is at rest under the action of given forces, obtain the equation which determines the pressure at any point of the fluid.

(b) A circular area of radius a is immersed with its plane vertical and centre at a depth h. Find the depth of the centre of pressure.

(c) If the absolute temperature T at a height z is a function $f(z)$ of the height, show that the ratio of the pressures at two heights z_1 and z_2 is given by

$$\log \frac{P_2}{P_1} = -\frac{g}{R} \int_{z_1}^{z_2} \frac{dz}{f(z)}$$

R being the constant in the equation $p = R\rho T$, where P_1, P_2 are the pressures at heights z_1 and z_2 respectively.

(d) Prove that a floating body is in stable or unstable equilibrium as the metacentre is above or below the centre of gravity of the body.

(e) Masses m, m' of two gases in which the ratios of the pressure to the density are respectively k and k' are mixed at the same temperature. Prove that the ratio of the pressure to the density in the compound is

$$\frac{mk + m'k'}{m + m'}$$

4. Answer *either* (a) *or* (b) :

(a) (i) If a fluid is at rest under the action of the forces X, Y, Z per unit mass, find the differential equations of the curves of equal pressure and density. 5

(ii) If the components parallel to the axes of the forces acting on the element of fluid at (x, y, z) be proportional to $y^2 + 2\lambda yz + z^2$, $z^2 + 2\mu zx + x^2$, $x^2 + 2\nu xy + y^2$, show that, if equilibrium is possible, we must have $2\lambda = 2\mu = 2\nu = 1$. 5

(b) (i) A mass of fluid rests upon a plane subject to a central attractive force $\frac{\mu}{r^2}$, situated at a distance c from the plane on the side opposite to that on which is the fluid. Show that the pressure on the plane is

$$\frac{\pi \rho \mu (a-c)^2}{a}$$

a being the radius of the sphere of which the fluid, on the plane in the form of a cap is a part. 5

(ii) If $X = y(y+z)$, $Y = z(z+x)$ and $Z = y(y-x)$, prove that surfaces of equal pressure are the hyperbolic paraboloids $y(x+z) = c(y+z)$ and the curves of equal pressure and density are given by $y(x+z) =$ constant and $y + z =$ constant. 5

5. Answer *either* (a) *or* (b) :

(a) (i) If an area is bounded by two concentric semi-circles with their common bounding diameter in the free surface, prove that the depth of the centre of pressure is

$$\frac{3\pi (a+b)(a^2+b^2)}{16(a^2+ab+b^2)}$$

where a and b are the radii of the outer and inner circles respectively. 5

(ii) A semi-ellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth. If the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure. 5

(b) (i) A hollow hemispherical shell has a heavy particle fixed to its rim, and floats in water with the particle just above the surface and with the plane of the rim inclined at an angle 45° to the surface. Show that the weight of the hemisphere is to the weight of the water which it would contain $4\sqrt{2}-5:6\sqrt{2}$. 5

(ii) A right cone is totally immersed in water, the depth of the centre of its base being given. Prove that p, p', p'' being the resultant pressures in its convex surface, when the sines of the inclination of its axis to the horizontal are s, s', s'' respectively

$$p^2(s'-s'')+p'^2(s''-s)+p''^2(s-s')=0$$ 5

6. Answer *either* (a) or (b) :

(a) (i) Show that for a perfect gas C_P is greater than C_V, where C_P is the specific heat keeping pressure as constant and C_V is the specific heat keeping volume as constant. 5

(ii) A gas satisfying Boyle's law $p = k\rho$ is acted on by the forces

$$X = -\frac{y}{x^2+y^2},\ Y = \frac{x}{x^2+y^2}$$

Show that the density varies as $e^{\theta/k}$, where $\tan\theta = \frac{y}{x}$. 5

(b) (i) Prove that a floating body is in stable or unstable equilibrium as the metacentre is above or below the centre of gravity of the body. 5

(ii) A cone of given weight and volume floats with its vertex downwards, prove that the surface of the cone in contact with the liquid is least when its vertical angle is

$$2\tan^{-1}\frac{1}{\sqrt{2}}$$ 5

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2019

MATHEMATICS

(Major)

Paper : 6.2

(Numerical Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) If $x = 2 \cdot 536$, find the absolute and the relative errors if x is truncated to two-decimal digits.

(b) Define 'truncation' error.

(c) Write the following numbers correct up to four significant figures :

0·00408, 0·10254

(d) Evaluate $\Delta^2\left(\frac{1}{x-1}\right)$, taking $h=1$.

(e) Show that $\Delta \cdot \nabla \equiv \Delta - \nabla$.

(f) State Lagrange's interpolation formula for $(n+1)$ unequally spaced arguments.

(g) Write Simpson's one-third rule in numerical integration.

2. Answer the following questions : 2×4=8

(a) Find the number of significant figures in 1·8921 given its relative error as $0\cdot 1\times 10^{-2}$.

(b) Prove that

$$\Delta \log f(x) = \log\left[1+\frac{\Delta f(x)}{f(x)}\right]$$

(c) Write the proper numerical differentiation formula to find the first derivative of a function $f(x)$ at a point x near the middle of a given set of tabulated values.

(d) Write a short note on numerical integration.

3. Answer the following questions : 5×3=15

(a) If $\Delta x = 0\cdot 005$, $\Delta y = 0\cdot 001$ be the absolute errors in $x = 2\cdot 11$ and $y = 4\cdot 15$, find the relative error in the computation of $x+y$.

(b) Use the method of separation of symbols to prove the following identity :

$$u_0 - u_1 + u_2 - \cdots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \cdots$$

Or

Find the missing entry in the following table :

x	0	1	2	3	4
y_x	1	3	9	—	81

(c) Find the cubic polynomial corresponding to the following data and hence evaluate $f(2\cdot 4)$:

x	0	1	2	3	4
$f(x)$	1	2	1	10	41

Or

Prove that the nth order divided differences of a polynomial of degree n in x are constants.

4. Answer any *one* part :

(a) (i) Apply Gauss's forward formula to find the value of u_9, if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{12} = 35$, $u_{16} = 40$.

(ii) From the following data, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1$:

x	1	2	3	4	5	6
y	1	8	27	64	125	216

5+5=10

(b) (i) From the following table, find the value of y when $x = 1·62$, using Stirling's formula :

x	1·3	1·4	1·5	1·6
$y = f(x)$	0·26236	0·33647	0·40547	0·47000

x	1·7	1·8	1·9
$y = f(x)$	0·53063	0·58779	0·64185

(ii) Calculate the value of $\int_0^1 \frac{x}{1+x}\,dx$ correct up to three significant figures taking six intervals, using the trapezoidal rule. 5+5=10

5. Answer any *one* part :

(a) (i) A rod is rotating in a plane about one of its ends. If the following table gives the angle θ (in radians) through which the rod has turned for different values of time t (in seconds), find its angular velocity at $t = 7$ sec :

t (in seconds)	0·0	0·2	0·4	0·6	0·8	1·0
θ (in radians)	0·0	0·12	0·48	1·10	2·0	3·20

(ii) Find the maximum and minimum values of y from the following data :

x	0	1	2	5
y	2	3	12	147

5+5=10

(b) (i) The velocity v of a particle moving in a straight line covering a distance x in time t are given by the following table :

x	0	10	20	30	40
v	45	60	65	54	42

Find the time taken to traverse the distance of 40 units.

(ii) Find the value of $\int_0^{0\cdot6} e^x dx$ by Simpson's $\frac{1}{3}$rd rule, dividing the range into six equal parts. 5+5=10

6. Answer any *one* part :

(a) (i) Give the geometrical interpretation of Newton-Raphson method.

(ii) Find the root of $\tan x + x = 0$, using bisection method, lying between 2 and 2·1. (Perform five iterations) 5+5=10

(Continued

(b) (i) Find a negative root of the equation $x^3 - \sin x + 1 = 0$ correct to three decimal places, using Newton-Raphson method.

(ii) Derive the rate of convergence of Newton-Raphson method. 5+5=10

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2019

MATHEMATICS

(Major)

Paper : 6.3

(Computer Programming in C)

Full Marks : 40

Time : 2 hours

The figures in the margin indicate full marks for the questions

1. Answer any *six* of the following as directed : 1×6=6

(a) ROM is a permanent storage medium. (State True or False)

(b) Library functions are included in the header file ____. (Fill in the blank)

(c) The output of $(3<2)||(4>3)$ is ____. (Fill in the blank)

(d) Justify why 7 days is not an identifier.

(e) Write the form of scanf( ) function.

(f) C language is a fourth-generation language.

(Write True or False)

(g) Which statement is used to exit from a statement block in the switch statement?

(h) What is a compiler?

2. Answer the following questions : 2×2=4

(a) What are an object program and a source program?

(b) Draw the flowchart to find the sum of given three numbers.

3. Answer any *two* of the following questions : 5×2=10

(a) Convert the following mathematical expressions into C expressions : 1+2+2=5

(i) $p = x^{\sqrt{y}} + y^{\sqrt{x}}$

(ii) $q = \sin a \cos b - |g - h| + \sqrt{ab} + \log(\cos x)^2$

(iii) $r = \sqrt{1 + x^3} + \dfrac{\log \cos 2x}{1 + |y|} + e^x$

(b) Write a C program to find the value of y using switch statement, where

$$y(x, n) = \begin{cases} 1 + x^n & \text{, when } n = 1 \\ 1 + 1/x^n & \text{, when } n = 2 \\ 1 + nx & \text{, when } n > 2 \text{ or } n < 1 \end{cases}$$

(c) Differentiate built-in functions and user-defined functions. Distinguish between local and global variables in C.

4. Answer any *four* of the following questions : 5×4=20

(a) Using two-dimensional array, write a C program to subtract two matrices of order 4×3.

(b) Write a note on conditional statement and its various types.

(c) Using recursive function, write a C program to find

$$^{n}C_{r} = \frac{\lfloor n}{\lfloor r \lfloor n - r}$$

(d) (i) What is a constant? Explain the constants used in C.

(ii) What is a variable? How are the variables declared in C? 3+2=5

(e) Using while loop, write a C program to generate and print Fibonacci series up to the term n.

(f) What are the various storage classes in C? Discuss their uses and scope.

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2019

MATHEMATICS

(Major)

Paper : 6.4

(Discrete Mathematics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) State division algorithm of integers.

(b) Write the least positive integer of the form $172x+20y$, $x, y \in \mathbb{Z}$.

(c) If p is a prime and $a \in \mathbb{Z}$, then show that $(a, p)=1$ or $p \mid a$.

(d) State the converse of Fermat's theorem. Is it valid?

(e) Write the value of the sum $\sum_{d|n} \mu(d)$.

(f) State Chinese remainder theorem.

(g) What is the geometrical interpretation of a Diophantine equation $f(x, y) = 0$?

2. Answer the following questions : 2×4=8

(a) Show that there is no positive integer n such that $0 < n < 1$.

(b) Show that $\phi(n)$ is even if $n > 2$.

(c) State and prove the converse of Wilson's theorem.

(d) If x, y, z are primitive, positive, Pythagorean triple, then show that

$$\left(\frac{z-x}{2}, \frac{z+x}{2}\right) = 1$$

where x is odd.

3. Answer the following questions : 5×3=15

(a) Let $a, b \in \mathbb{Z}$, a or $b \neq 0$, $G = (a, b)$. Show that $G = ax_0 + by_0$ for some $x_0, y_0 \in \mathbb{Z}$.

Or

Show that every integer $n > 1$ can be expressed as a product of primes. Find the prime factorization of 40! 3+2=5

(b) Using Chinese remainder theorem, find the least positive integer which leaves the remainders 1, 6, 2 when divided by 7, 10, 11 respectively.

Or

Let $n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$ be the prime factorization of a positive integer $n > 1$. Show that the positive divisors of n are precisely the integers of the form $d = p_1^{a_1} p_2^{a_2} \cdots p_r^{a_r}$, where $0 \le a_i \le k_i$, $i = 1, 2, \cdots, r$.

(c) The linear congruence $ax \equiv b \pmod{m}$ has a solution if and only if $(a, m) \mid b$. Prove it.

Or

Show that the Diophantine equation $x^4 + y^4 = z^2$ has no solutions in positive integers.

4. Answer *either (a) or (b)* : 10

(a) (i) Let p be a prime and $\gcd(a, p) = 1$. Then show that the congruence $ax \equiv y \pmod{p}$ has a solution x_0, y_0 such that

$$0 < |x_0| < \sqrt{p}, \ 0 < |y_0| < \sqrt{p}$$ 5

(ii) If p_n is the nth prime number, then show that the sum

$$\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_n}$$

is never an integer. 5

(b) (i) Show that Euler's function ϕ is a multiplicative function. 7

(ii) Show that no prime of the form $4k+3$ is a sum of two squares. 3

5. Answer *either (a) or (b)* : 10

(a) (i) Give an example of an infinite Boolean algebra. In a Boolean algebra B, show that

$$a + a = a, \ a \cdot (a + b) = a, \ a, b \in B$$ 2+3=5

(ii) State and prove the 'principle of duality' in a Boolean algebra. Write down the dual of the proposition $a + b = 0 \Leftrightarrow a = 0, \ b = 0$. 4+1=5

(b) (i) Simplify the Boolean expression

$$(x + y)(x + z)(x' y')'$$

Draw a switching circuit which realizes the Boolean expression $x + y(z + x'(t + z'))$. 3+2=5 *(ii)* Construct the switching table for the switching function f represented by the Boolean expression $xyz + x'(y+z)$. 5

6. Answer *either (a) or (b)* : 10

(a) *(i)* Define 'logical equivalence'. Prove that if $\vDash A$ and $\vDash A \to B$, then $\vDash B$. 1+4=5

(ii) Construct the truth tables for NOR($\downarrow$) and NAND($\uparrow$). Show that $\{\wedge, \to\}$ is not an adequate system of connectives. 2+3=5

(b) *(i)* Using principle of substitution, show that if A, B be any two statement formulae, then

$$A \to B \equiv \sim B \to \sim A$$ 5

(ii) Assuming the truth value of $p \to q$ be T, construct the truth table for $(\sim p \wedge q) \leftrightarrow (p \vee q)$. 2

(iii) Define a truth function. Construct the truth function generated by the statement formula

$$\sim(\sim p \wedge q)$$ 1+2=3

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2019

MATHEMATICS

(Major)

Paper : 6.5

(Graph and Combinatories)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : $1\times7=7$

(a) Write sum rule principle of combinatories.

(b) A farmer buys 3 cows, 2 pigs and 4 hens from a man who has 6 cows, 5 pigs and 8 hens. How many choices does the farmer have?

(c) Suppose 6 people randomly arrive at a darkroom to retrieve their hats. Find the number of ways such that no person picks his own hat.

(d) A tree

(i) is always a disconnected graph

(ii) is always a connected graph

(iii) may be connected or disconnected

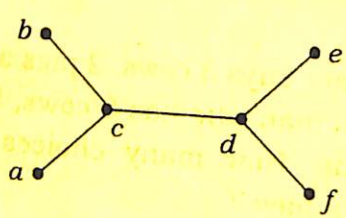
(iv) None of the above

(Choose the correct answer)

(e) Define simple graph.

(f) How many degrees of each vertex are there in a circuit?

(g) Find the eccentricity of the vertex a of the following graph :



2. Answer the following questions : $2 \times 4 = 8$

(a) There are 9 students in a class. Find the number of ways that the 9 students can take 3 different tests, if 3 students are to take each test.

(b) Represent the graph $G(V, E)$, where the vertex set V and edge set E are as follows :

$$V = \{1, 2, 3, 4\},\ E = \{(x, y) : |x - y| \leq 1,\ x, y \in V\}$$

(c) Let v be a point of a connected graph G. If there exist points u and w distinct from v such that v is in every u–w path, then show that v is a cut point of G.

(d) Prove that every non-trivial connected graph has at least two points which are not cut points.

3. Answer the following questions : $5 \times 3 = 15$

(a) Show that $C(2n, 2) = 2C(n, 2) + n^2$.

(b) Prove that a simple graph with at least two vertices has at least two vertices of same degree.

Or

Prove that the sum of the degree of all vertices of a graph is an even integer.

(c) Let G be a simple graph with at most $2n$ vertices. If the degree of each vertex is at least n, then show that the graph is connected.

Or

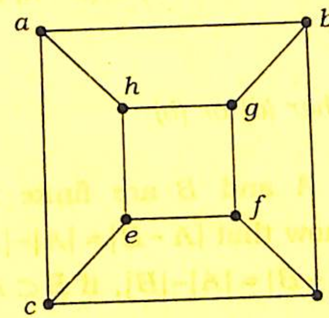
Prove that every tree has a centre consisting of either one point or two adjacent points.

4. Answer *either (a) or (b)* :

(a) Prove that a graph with at least $2n$ points is n-connected if and only if for any two disjoint sets V_1 and V_2 of n points each, there exists n disjoint path joining these two sets of points. 10

(b) (i) Show that the vertex connectivity of any graph G is always less than or equal to edge connectivity of G. 6

(ii) Find the edge connectivity and vertex connectivity of the graph G represented by



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5. Answer *either (a) or (b)* :

(a) If G is a simple graph with number of vertices $n(\geq 3)$ and if $\deg(v)+\deg(w)\geq n$ for any pair of non-adjacent vertices v and w, then show that G is Hamiltonian. 10

(b) (i) If G is a simple graph with n vertices and e edges with $n \geq 3$ and $e \geq \frac{1}{2}(n^2 - 3n + 6)$, then show that G is Hamiltonian. 5

(ii) If G is a Hamiltonian graph, then show that any non-empty proper subset S of V, $w(G-S) \leq |S|$. 5

6. Answer *either (a) or (b)* :

(a) (i) If A and B are finite sets, then show that $|A-B| = |A| - |A \cap B|$ and $|A-B| = |A| - |B|$, if $B \subset A$. 3+2=5

(ii) How many solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 16$, where each $x_i \geq 27$? 5

(b) (i) There are five different Hindi books, six different English books and eight different Sanskrit books. How many ways are there to pick two books not both in the same language? 4

(Continued)

(ii) How many solutions does $x_1 + x_2 + x_3 = 13$ have where x_1, x_2, x_3 are non-negative with $x_1 \leq 4$, $x_2 \leq 5$ and $x_3 \leq 6$? 6

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