2016

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Group—A

(Mathematical Methods)

(*Marks* : 30)

1. Answer the following questions : 1×4=4

   (a) Find the real part of $\frac{1+z}{1-z}$.

   (b) What is the argument of $-3i$?

   (c) Define pole and residue.

   (d) Find the principal value of i^i.

2. (a) Find the complex conjugate of the functions

$$(x+iy)\cdot(a+ib) \text{ and } \frac{x-iy}{a+ib}$$

   where x, y, a and b are real. 4

   (b) Obtain the modulus of the complex number $\frac{1-i}{1+i}$. 2

(Turn Over)

3. (a) State De Moivre's theorem. 2

(b) Using De Moivre's formula, evaluate 2

$$(\cos 20° + i\sin 20°)^9$$

4. (a) Define equivalent contour. 2

(b) Verify if the function $f(z) = z$ is analytic. 4

Or

(i) Determine if the function e^{iz} is analytic. 2

(ii) Prove that 2

$$\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$$

5. (a) Check the analyticity and hence find the derivative of the function $f(z) = \sin z$. 5

Or

Find Taylor series expansion about the origin for $f(z) = \ln(1+z)$.

(b) Find Laurent expansion for the function $f(z) = \dfrac{\sin z}{z^4}$ about $z_0 = 0$ and hence classify the singularity and calculate the residue.

Or

State and prove Cauchy's integral theorem.

(Continued)

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6. Answer the following questions : 1×3=3

(a) State the principle of virtual work.

(b) Define central force and write down a general expression for it.

(c) Define Hamiltonian of a system.

7. Answer any *three* of the following questions : 2×3=6

(a) Explain with example the meaning of holonomic constraint.

(b) Show that angular momentum is a constant of central force motion.

(c) In a two-body system, the masses are in the ratio 4 : 1. The mass of the lighter body is 10^{-28} g. Estimate the reduced mass of the system.

(d) What are generalized coordinates and generalized velocities?

8. Answer any *four* of the following questions : 4×4=16

(a) Show that a two-body central force problem can be reduced to one-body problem.

(Turn Over)

(b) Find the equation of motion of a system with the given Lagrangian

$$L = \frac{1}{2} e^{\alpha t} (\dot{x}^2 - \omega^2 x^2)$$

where α and ω are constants.

(c) Obtain the general differential equation of a cental orbit.

(d) Show that if the Lagrangian function does not contain the time explicitly, the total energy of the conservative system is conserved.

(e) Construct the Lagrangian and hence equation of motion of a simple pendulum placed in a uniform gravitational field.

(f) Set up Lagrangian equation for an Atwood machine and find an expression for its acceleration.

9\. Establish Hamilton's canonical equations. 5

Or

Obtain Lagrange's equation of motion for a conservative system using D'Alembert's principle. 5

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2016

PHYSICS

(Major)

Paper : 5.3

(Quantum Mechanics and Astrophysics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

rite the answers to the two Groups in separate books

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Answer any *four* questions as directed : 1×4=4

(a) Mention one experiment which demonstrates the wave nature of matter.

(b) Calculate the energy of a radiowave photon of wavelength $\lambda \approx 10$ m.

(Turn Over)

(c) Which one of the following is a true statement for photoelectric effect?

(i) Kinetic energy of ejected electrons depends on the intensity of incident light.

(ii) Kinetic energy of ejected electrons depends linearly on the frequency of an incident light.

(iii) Kinetic energy of ejected electrons is always zero.

(iv) It proves wave nature of light.

(Choose the correct option)

(d) Show that de Broglie wavelength of a relativistic particle of mass m moving with velocity v can be approximated as

$$\lambda \approx \frac{h}{mv} - \frac{hv}{2mc^2}$$

(e) The uncertainty of position of a particle of mass m inside a black hole of mass M is about $\Delta x \approx 2GM/c^2$. Here G is Newton's gravitational constant and c is the velocity of light. Calculate the approximate energy of the particle.

(Continued)

2. Answer any *two* questions : 3×2=6

(a) Calculate the Compton wavelength for electron. An X-ray photon of wavelength $\lambda_0 = 1$ Å is incident on a free electron which is initially at rest. The photon is scattered at angle $\theta \simeq 30°$ from the initial direction. Also calculate the final wavelength of the photon. Given, electron mass $m \simeq 9 \cdot 1 \times 10^{-31}$ kg. 1+2=3

(b) The photoelectric work function W for lithium is 2·3 eV. Find the threshold frequency ν_t. If ultraviolet light of wavelength $\lambda = 3000$ Å is incident on a lithium surface, calculate the maximum kinetic energy of the photoelectrons. 1+2=3

(c) Energy of a particle of mass m moving in the gravitational field of a mass M is given by

$$E = \frac{p^2}{2m} - \frac{GMm}{r}$$

Write down the time-independent Schrödinger equation for this system. The wave function of a particle at r is defined as $\psi(r) = f\frac{e^{ikr}}{r}$.

Obtain the probability of finding the particle at r. Here f is a constant complex number. 2+1=3

(Turn Over)

3. Answer any *four* questions : 5×4=2

(a) The energy distribution of blackbody radiation is given by Planck's law

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Show that for long wavelength

$$\rho(\lambda, T) \to \frac{8\pi kT}{\lambda^4}$$

and for short wavelength

$$\rho(\lambda, T) \to \frac{8\pi hc}{\lambda^5} \exp\left\{-\frac{hc}{\lambda kT}\right\}$$

What is Planck's quantum hypothesis? Mention one experiment for determining Planck's constant, h. 1½+1½+1+1

(b) Show that the ratio of kinetic energy of an alpha particle of mass m_α to that of a proton of mass m_p having same de Broglie wavelength of 1 Å is

$$\frac{KE_\alpha}{KE_p} = \frac{m_p}{m_\alpha}$$

Thermal neutrons in a nuclear reactor have kinetic energy of $3kT/2$, where k is the Boltzmann constant and T is the absolute temperature. What is the energy in eV and de Broglie wavelength of a thermal neutron at room temperature $T \simeq 300$ K? 2+1+2=5

(c) What is the basic difference between Davisson-Germer experiment and G. P. Thomson's experiment on electron diffraction? Show that de Broglie wavelength of an electron of mass m and charge e, accelerated by potential V is

$$\lambda = \frac{h}{\sqrt{2meV}}$$

What is the significance of electron diffraction experiments? Give example of a phenomenon which demonstrates particle nature of light. 1+2+1+1=5

(d) The energy of a particle of mass m moving in a gravitational field of mass M is given by

$$E = \frac{p^2}{2m} - \frac{GMm}{r}$$

(Turn Over

Suppose radius of the orbit satisfies the uncertainty principle $rp \simeq \hbar$. Show that energy of the particle becomes

$$E = \frac{\hbar^2}{2mr^2} - \frac{GMm}{r}$$

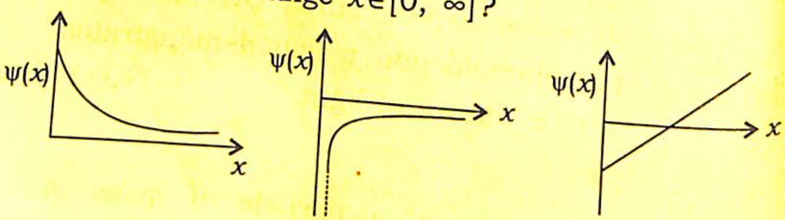
Also, show that this energy has a minimum at $r = r_0 = (\hbar^2 / GMm^2)$. Obtain the minimum value of energy E_{min}.

1+2+2=5

(e) Briefly discuss G. P. Thomson's experiment of electron diffraction, and its significance for quantum theory. 5

4. Answer any *two* questions : 5×2=10

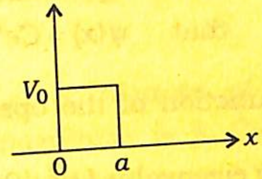
(a) Which one of the following graphs represents a well-behaved wave function in the range $x \in [0, \infty]$?



Find out the normalization constant A for the wave-function $\psi(x) = A \sin\left(\frac{n\pi x}{2a}\right)$ in the range $-a \le x \le a$.

1+4=5

(b) Consider a rectangular potential barrier as shown in the figure



so that $V(x) = \begin{cases} 0, & x < 0 \\ V_0, & 0 < x < a \\ 0, & x > a \end{cases}$

The wave functions in the external regions are given by

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx}, & x < 0 \\ Ce^{ikx}, & x > a \end{cases}$$

(i) Identify incident wave, reflected wave and transmitted wave if the particle comes from the left side.

(ii) Show that the probability current densities are

$$j = \begin{cases} \frac{\hbar k}{m}(|A|^2 - |B|^2), & x < 0 \\ \frac{\hbar k}{m} |C|^2, & x > a \end{cases}$$

(iii) Write down the expression for reflection coefficient R and transmission coefficient T. 1+3+1=

(c) How do you represent dynamical variables in quantum mechanics? Show that $\psi(x) = Ce^{ipx/\hbar}$ is an eigenfunction of the operator $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$ having eigenvalue $(p^2/2m)$. What is the form of the wave function for a free particle? 1+3+1=5

GROUP—B

(Astrophysics)

(*Marks* : 20)

5. Answer any *three* from the following : 2×3=6

(a) What is the angle of inclination between the ecliptic and the celestial equators? A star has right ascension $\alpha = 6^h 51^m$ and another star has $\alpha = 12^h 52^m$. Which one of them rises earlier? 1+1=2

(b) Draw a neat diagram of the celestial sphere showing a star in northern hemisphere, the celestial equator, hour angle, observer's meridian and the right ascension of the star. 2

(c) The parallax angle of the star 61 Cygni is 0″.285. Calculate its distance. 2

(d) A star is at a distance of 4 pc. Its apparent magnitude is 2. Calculate its absolute magnitude. 2

(e) Define local sidereal time. If right ascension (α) and declination (δ) of a star are given as $(18^h 51^m 12^s, -05°08'01'')$, which hemisphere should be chosen for observing the star? 1+1=2

6. Answer any *two* of the following : 4×2=8

(a) What is the basis of H-D classification of stars? Display the ranges of surface temperatures of *O*-, *B*- and *G*-type stars. In the HR diagram, justify how stars with 'extremely low temperature but large luminosity' and stars with 'extremely high temperature but low luminosity' can exist. 1+1+2=4

(Turn Over

(b) Out of pp-chain and CNO cycle, which one dominates the energy production in sun-like stars? What happens to the core of a star once hydrogen burning is exhausted? Discuss how a red giant forms. 1+1+2=4

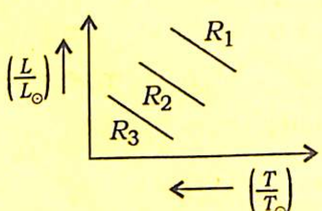
(c) Show that luminosity (L), radius (R) and surface temperature (T) of a star are related as

$$\left(\frac{L}{L_\odot}\right)=\left(\frac{R}{R_\odot}\right)^2\left(\frac{T}{T_\odot}\right)^4$$

where $L_\odot$, $R_\odot$ and $T_\odot$ are corresponding quantities for the sun.

The adjacent figure shows HR diagram with three straight lines across the main sequence, representing stars of radii R_1, R_2 and R_3. Which of the three lines represents the biggest stars?

Show the evolutionary track of a sun-like and a massive, luminous star in the HR diagram. 1+1+2=4



7. Write short notes on any *two* of the following : 3×2=6

(a) Spectral classification of stars

(b) Celestial coordinates

(c) PP-chain and CNO cycle

(d) Trigonometric parallax

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2016

PHYSICS

(Major)

Paper : 5.4

(Electronics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions briefly : 1×7=7

(a) Why are semiconductor diodes called non-linear device?

(b) What is the condition that must be satisfied in order to receive maximum power by a two-terminal network from another network?

(c) What should be the biasing of emitter-base and collector-base junctions of a transistor to operate it in active region?

(d) In class A transistor amplifier, what proportion of the input-cycle the transistor conducts?

(e) What should be the value of input resistance of an ideal operational amplifier?

(f) What is the cut-off frequency beyond which the ionosphere does not reflect electromagnetic waves?

(g) In AM transmission, what proportion of total power is carried away by the carrier wave for 100% depth of modulation with 600 watts of total power?

2. Answer the following questions : 2×4=8

(a) Distinguish between Zener breakdown and avalanche breakdown in semiconductor diodes.

(b) What could be the possible reasons for reduction in voltage gain of transistor *R-C* coupled amplifier at high frequency?

(Continued)

(c) What do you understand by sequential circuits? Give one example.

(d) What are the relative advantages and disadvantages of negative feedback in transistor amplifiers?

3. Draw the circuit diagram of a common-emitter transistor amplifier with self- or emitter-biasing configuration. State the advantages of self-biasing circuit. Derive the relationship between common-emitter current amplification factor (β) and common-base current amplification (α). 1+1+3=5

4. Explain why half-wave rectifier is called a poor device for rectification. Derive an expression for efficiency of such rectifier. 2+3=5

Or

Draw the symbolic representation with proper biasing for each of the following electronic devices when in operation :

(a) Zener diode

(b) Varactor diode

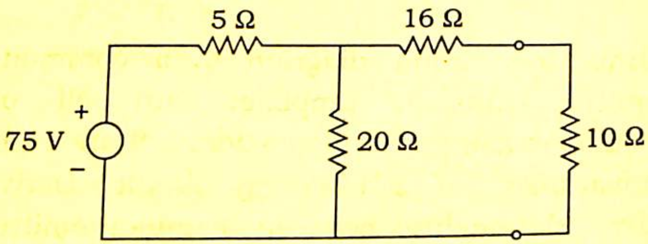
(c) LED

(d) Photodiode

How does LED differ from ordinary *p-n* junction diode? 5

(Turn Over)

5. Transform the following circuit into Thevenin's equivalent circuit and hence find the value of *(a)* Thevenin's equivalent impedance, *(b)* Thevenin's equivalent voltage source, and *(c)* load current and power in 10 Ω resistance : 5



6. Answer any *two* questions from the following : 5×2=10

(a) Derive the *h*-parameters of transistor. Draw the *h*-parameter equivalent circuit of transistor CE amplifier.

(b) Draw the block diagram of a feedback amplifier and find an expression for overall gain of such amplifier. Define positive and negative feedback.

(c) What are the fundamental differences in operation of class A, class B and class C amplifiers explaining with the help of dynamic curves. Which one of them has maximum power conversion efficiency? Draw the circuit diagram of a push-pull amplifier. 3+1+1=5

(Continued)

(d) The open-loop gain of a feedback amplifier is 200 and feedback factor is $\beta = 0 \cdot 5$. Assuming negative feedback, determine—

(i) desensitivity factor (*D*);

(ii) close-loop gain.

If the open-loop gain changes by +10%, find the percentage (%) change in the close-loop gain and its value. 1+1+3=5

7. Answer any *two* questions of the following : 5×2=10

(a) What are the characteristics of an ideal operational amplifier? Explain the concept of virtual ground in an operational amplifier. Draw the basic integrator and differentiator circuits of operational amplifier. 2+2+1=5

(b) Give the basic non-inverting circuit of an operational amplifier and derive the expression for voltage gain of such circuit. Find the output of the non-inverting operational amplifier for $V_{\text{in}} = 5 \cdot 5$ mV, $R_f = 90$ kΩ and $R_1 = 10$ kΩ. 1+2+2=5

(Turn Over)

(c) State why NAND and NOR gates are called universal gate. Give the truth table of NAND and NOR gates. Draw the diagram to show how OR, AND and NOT gates can be constructed using NAND gates only. 1+1+3=5

(d) Convert the decimal numbers 256·50 and 128·25 to its binary equivalent and find the difference using 2's complement method. Add the binary numbers 1011·10 and 111·01. Verify the result by converting them to decimal equivalent. 2+2+1=5

(e) Define FSK and PSK methods of digital transmission. Draw the block diagram of any one of them. 4+1=5

8. Answer any *two* questions of the following : 5×2=10

(a) Draw the block diagram of the analog communication system. State with the help of diagram any one method of generation of AM signal. Write the advantages of FM transmission over AM transmission. 1+2+2=5

(Continued)

(b) A 400 watts carrier is amplitude modulated to a depth of 100%. Calculate the total power in case of AM and SSB techniques. How much power saving (in watts) is achieved for SSB compared to AM? 2+2+1=5

(c) (i) Draw the block diagram of super-heterodyne AM receiver.

(ii) The instantaneous voltage of an AM wave is given by

$$V_{AM} = 20(1 + 0\cdot 5 \sin 2\pi \times 10^3 t) \sin 2\pi \times 10^6 t \text{ volts}$$

Find the amplitude and frequency of two sidebands. 2+3=5

(d) Write short note on any *one* of the following : 5

(i) Norton's theorem

(ii) Multivibrator

(iii) *R-S* flip-flop

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