Total number of printed pages–8

3 (Sem–6) MAT M1

2020

MATHEMATICS

(Major)

Paper : 6·1

(*Hydrostatics*)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Which of the following is a false statement?

Pressure of a fluid at a given depth

(i) is equal in all directions

(ii) is dependent on the shape of the container

(iii) depends on the density of the fluid

(iv) acts in the normal direction to the surface with which it is in contact

Contd.

(b) If two volumes v_1, v_2 of different gases at pressures p_1 and p_2 and absolute temperatures T_1 and T_2 are mixed together so that the volume of the mixture is V and the absolute temperature T, then what is the pressure of the mixture?

(c) The amount of heat required to raise the temperature of a body by one degree is called

(i) thermal capacity

(ii) specific heat

(iii) isothermal temperature

(iv) None of the above

(d) Write an equation that expresses the relation between pressure and volume in adiabatic change.

(e) Define centre of pressure of a plane area immersed in a liquid at rest.

(f) If the equilibrium is unstable, what is the position of the metacentre w.r.t. the centre of gravity of the body?

(g) Which of the following is true?

Fluids —

(i) cannot be compressed

(ii) are not affected by change in temperature

(iii) are not viscous

(iv) do not offer any resistance to change in shape

2. Answer the following: 2×4=8

(a) Obtain the expression for pressure at a point in a homogeneous liquid of density ρ and acted upon by component forces per unit mass X, Y, Z.

(b) Obtain the formula for determination of the centre of pressure of any plane area in Cartesian coordinate.

(c) Define metacentre and metacentric height of a floating body.

(d) Find the work done in compressing a gas under isothermal conditions, where π is the atmospheric pressure.

3. Answer ***any three*** parts: 5×3=15

(a) A solid cone is following with its axis vertical and vertex downwards. Discuss its stability of equilibrium.

(b) Show that in a conservative field of force, the surface of equipressure, equidensity and equipotential energy coincide.

(c) A quadrant of a circle is just immersed vertically with one edge in the surface, in a liquid, the density of which varies as the depth. Find the centre of pressure.

(d) A solid body consists of a right cone joined to a hemisphere on the same base and floats with the spherical portion partly immersed. Prove that greatest height of the cone consistent with stability is $\sqrt{3}$ times of the radius of the base.

(e) For a perfect gas establish the relation $C_p - C_v = R$, where symbols have their usual meanings.

4. Answer ***any one*** part :

(a) (i) Show that the forces represented by $X = \mu\left(y^2 + z^2 + yz\right)$,

$$Y = \mu\left(z^2 + x^2 + zx\right),$$

$$Z = \mu\left(x^2 + y^2 + xy\right)$$

will keep a mass of liquid at rest, if the density $\propto \frac{1}{(\text{distance})^2}$ from the plane $x + y + z = 0$; and the curves of equal pressure and density will be circles. 5

(ii) A circular cylinder of radius a, is floating freely in water with axis vertical. At first the water was at rest and it is then made to rotate about an axis, which is the axis of the cylinder, with an angular velocity of ω. Show that in the second case an extra length $\frac{\omega^2 a^2}{4g}$ of the surface of the cylinder is wetted. 5

(b) (i) A given volume of liquid is at rest on a fixed plane under the action of a force, to a fixed point in the plane, varying as the distance. Find the pressure at any point in the liquid and the whole pressure on the fixed plane. 5

(ii) A tube in the form of a parabola held with its vertex downwards and axis vertical, is filled with two different liquids of densities δ and δ'. If the distances of the free surfaces of the liquids from the focus be r and r' respectively, show that the distance of their common surface from focus is

$$\frac{r\delta - r'\delta'}{\delta - \delta'}.$$ 5

5. Answer ***any one*** part :

(a) (i) Show that the depth of the centre of pressure of the area included between the arc and the asymptote of the curve $(r-a)\cos\theta = b$ is $\frac{a}{4} \times \frac{3\pi a + 16b}{3\pi b + 4a}$, the asymptote being in the surface and the plane of the curve vertical. 5

(ii) A hemispherical bowl is filled with water. Find the horizontal thrust on one-half of the surface divided by a vertical diameter plane and show that it is $\frac{1}{\pi}$ times the magnitude of the resultant fluid thrust on the whole surface. 5

(b) (i) A solid displaces $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ of its volumes, respectively when it floats in three different liquids. Find what fraction of its volume it displaces when it floats in a mixture formed of equal volumes of three liquids. 5

(ii) A quadrant of a circle is just increased vertically with one edge in the surface, in a liquid, the density of which varies as the depth. Determine the centre of pressure. 5

6. Answer ***any one*** part :

(a) (i) A solid cone of semivertical angle α, specific gravity σ floats in equilibrium in the liquid of specific gravity ρ with its axis vertical and vertex downwards. Show that the equilibrium is stable if $\frac{\sigma}{\rho} > \cos^6 \alpha$. 5

(ii) A bent tube of uniform bore, the arms of which are at right angles, revolves with constant angular velocity ω about the axis of one of its arms, which is vertical and has its extremity immersed in water. Prove that the height to which the water will rise in the vertical arm is $\frac{\pi}{ge}\left[1 - e^{\frac{-\omega^2 a^2}{2k}}\right]$, a being the length of the horizontal arm, π the atmospheric pressure, ρ the density of water and k the ratio of the pressure of the atmosphere to its density. 5

(b) (i) The height of the Torricellian vacuum in a barometer is a inches and the instrument indicates a pressure of b inches of mercury when the true reading is c inches. If the faulty readings are due to an imperfect vacuum, prove that the true reading corresponding to an apparent reading of d inches is

$$d + \frac{a(c-d)}{a+b-d}.$$ 5

(ii) In convective equilibrium, show that the relation between absolute temperature T and height of the homogeneous atmosphere H is

$$T = T_0\left[1 - \left(\frac{\gamma - 1}{\gamma}\right)\frac{Z}{H}\right],$$

when the acceleration due to gravity is constant. 5

2020

MATHEMATICS

(Major)

Paper : 6·2

(Numerical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) If $\pi = \frac{22}{7}$ is approximated as 3·14, find the relative error and relative percentage error.

(b) Define 'absolute error'.

(c) Find the difference $\sqrt{2{\cdot}01} - \sqrt{2}$, correct to three significant figures.

(d) If m and n are positive integers, then show that $\Delta^m \Delta^n f(x) = \Delta^{m+n} f(x)$.

(e) Evaluate $\Delta^n\left(\frac{1}{x}\right)$, with 1 as the interval of differencing.

(f) Give the relationship between the operator Δ and the differential operator D.

(g) Write the general quadrature formula in numerical integration.

2. Answer the following questions : 2×4=8

(a) Find the number of significant figures in $x = 0.3941$ whose absolute error is 0.25×10^{-2}.

(b) Given $u_0 = 3$, $u_1 = 12$, $u_2 = 81$, $u_3 = 200$, $u_4 = 100$ and $u_5 = 8$, find $\Delta^5 u_0$.

(c) What is numerical differentiation ? Explain briefly its importance.

(d) Derive trapezoidal rule from Newton-Cotes quadrature formula.

3. Answer the following questions : 5×3=15

(a) Find the relative error for evaluation of $u = x_1 x_2$ with $x_1 = 4.51$, $x_2 = 8.32$ having absolute errors $\Delta x_1 = 0.01$ in x_1 and $\Delta x_2 = 0.01$ in x_2.

(b) Using the method of separation of symbols, prove the following :

$$(u_1 - u_0) - x(u_2 - u_1) + x^2(u_3 - u_2) - \ldots\ldots$$

$$= \frac{\Delta u_0}{1+x} - x\frac{\Delta^2 u_0}{(1+x)^2} + x^2\frac{\Delta^3 u_0}{(1+x)^3} - \ldots\ldots$$

Or

Find the function whose first difference is $9x^2 + 11x + 5$.

(c) A second degree polynomial passes through the points (1, −1), (2, −1), (3, 1) and (4, 5). Find the polynomial.

Or

Using Lagrange's interpolation formula, find the form of the function given by :

x :	3	2	1	−1
$f(x)$:	3	12	15	−21

4. Answer ***any one*** part :

(a) (i) Apply Stirling's formula to find a polynomial of degree 4 which takes the following tabular values :

x :	1	2	3	4	5
$y=f(x)$:	1	−1	1	−1	1

(ii) Using Newton's divided difference formula, construct the interpolating polynomial and hence compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=5$ using the following data :

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

5+5=10

(b) (i) Use Bessel's formula to find $y(0{\cdot}12)$ from the following data :

:	0	0·05	0·1	0·15	0·2	0·25
:	0	0·10017	0·20134	0·30452	0·41075	0·52110

(ii) Find the value of $\int_1^5 \log_{10} x\, dx$, taking 8 subintervals, by trapezoidal rule. 5+5=10

5. Answer ***any one*** part :

(a) (i) In a machine a slider moves along a fixed straight rod. Its distance x *cms* along the rod is given below for various values of time t seconds. Find the velocity and acceleration of the slider when $t=0{\cdot}3$.

t(sec) :	0	0·1	0·2	0·3	0·4	0·5	0·6
x(cm) :	30·13	31·62	32·87	33·64	33·95	33·81	33·24

(ii) The velocity $v\,(km/min)$ of a car which starts from rest, is given at fixed intervals of time t (*min*) as follows :

t :	2	4	6	8	10	12	14	16	18	20
v :	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 minutes. 5+5=10

(b) (i) Using Lagrange's formula and the following table, find $f'(3)$ and $f'(4)$:

x	: 1	2	4	8	10
$f(x)$	: 0	1	5	21	27

(ii) Find an approximate value of $\log_e 7$ using Simpson's rule to the integral $\int_1^7 \frac{dx}{x}$.

5+5=10

6. Answer *any one* part :

(a) (i) Derive the rate of convergence of the Secant method.

(ii) Compute the root of $e^x - 3x = 0$, using bisection method, lying between 1·5 and 1·6, correct to two decimal places. 5+5=10

(b) (i) Using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$, which is nearer to $x=2$, correct to three decimal places.

(ii) Find an approximate root of the equation $x^3 + x - 1 = 0$ near $x = 1$, by the Regula-Falsi method, correct to two decimal places.

5+5=10

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3 (Sem–6) MAT M 3

2020

MATHEMATICS

(Major)

Paper : 6·3

(Computer Programming in C)

Full Marks : 40

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any six*** questions : 1×6=6

(a) What is a source program ?

(b) What is an algorithm ?

(c) What is meant by 'machine level language'?

Contd.

(d) State True **or** False :

"C is a case sensitive language."

(e) $i+=1$ can also be written as ______ . *(Fill in the blank)*

(f) Which header file has to be included to use getch ( ) ?

(g) The ______ statement is used to transfer the control to the beginning of the loop. *(Fill in the blank)*

(h) Give an example of a volatile memory.

2. Answer ***any two*** questions : 2×2=4

(a) Write the following mathematical expression in C language :

$$e^{x+y} - \sin\left(\sqrt{|x+ny|}\right) + \log 2a$$

(b) What is an array variable ? How does it differ from an ordinary variable ?

(c) What are library functions ? Mention *any two* library functions of C available in math.h.

(d) Discuss the difference between assignment and equality with examples.

3. Answer ***any two*** of the following : 5×2=10

(a) What is a variable ? How is it declared ? What are the basic datatypes ? How does the type float differ from double in C language.

(b) Write a C program to add 3 × 3 matrices.

(c) Explain the utility of break and continue statement with the help of suitable examples.

4. Answer ***any two*** of the following : 5×2=10

(a) What is meant by a recursive function or recursion ? Using a recursive function, write a C program for computing the value of x^n for given values of x and n.

(b) Discuss the following operators with examples :

(i) Relational operator

(ii) Logical operator

(iii) Ternary operator.

(c) Give the general syntax of the if-else statement. Use it to write a C program to find the largest of three given numbers.

5. Answer ***any two*** questions :

(a) Explain briefly what is meant by 'call by value' and 'call by reference' of a function. 5

(b) Give the general form of 'for loop' in C. Explain how it works with the help of a suitable example. 5

OR

Give the general form of a function declaration. How is a function called? What are actual and formal arguments? 2+1+2=5

(c) Write a C program to sort a set of 10 given numbers in ascending order. 5

OR

Write a C program to find the roots of the quadratic equation $ax^2 + bx + c = 0$ for given a, b, c.

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3 (Sem–6) MAT M 4

2020

MATHEMATICS

(Major)

Paper : 6·4

(Discrete Mathematics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

. Answer the following questions as directed : $1\times7=7$

(a) Show that for any integer n, 1 divides n.

(b) If $\tau(n)$ is odd for an integer $n > 1$, then

(i) n is odd

(ii) n is even

(iii) n is a perfect square

(iv) n is a perfect square or twice a perfect square

(Choose the correct option)

(c) Give example of two integers a and b such that

$$a^2 \equiv b^2 \pmod 3 \text{ but } a \not\equiv b \pmod 3$$

(d) State Fermat's Little Theorem (FLT_1).

(e) Find the number of positive divisors of 7056.

(f) Write the absorption laws of propositional logic.

(g) Express 1225 as a sum of two squares.

2. Answer the following questions : 2×4=8

(a) If two co-prime integers a and b are such that a/c and b/c, then show that ab/c. Is this true when a and b are not co-prime? 1+1=2

(b) Find the remainder when 2356710825 is divided by 37.

(c) Express in disjunctive normal form :

$$1 + x_2' x_1'$$

(d) If $f(n) = \prod_{d/n} g(d)$, then show that

$$g(n) = \prod_{d/n} [f(d)]^{\mu\left(\frac{n}{d}\right)}.$$

3. Answer **any three** questions : 5×3=15

(a) If $a, b \in Z$, then show that a positive integer 'p' is a prime if and only if

$$p/ab \Rightarrow p/a \text{ or } p/b$$

(b) If (x, y, z) is a primitive solution of $x^2 + y^2 = z^2$, then show that one of x and y is even and the other is odd.

(c) If x and y are real numbers such that

(i) $[x+y] = [x] + [y]$ and

(ii) $[-x-y]=[-x]+[-y]$, then show that one of x or y is an integer and conversely.

(d) Show that a complete DNF is identically 1.

(e) Show that if $a_1, a_2, ..., a_k$ form a $RRS\ (mod\, m)$ then $k=\phi(m)$.

4. Answer **either** [(a) and (b)] **or** [(c) and (d)] :

(a) If a and b are positive integers then prove that :

$$gcd\,(a,b)\times lcm[a,b]=ab$$ 5

(b) If eggs are taken out from a basket two, three, four, five and six at a time, there are left over respectively one, two, three, four and five eggs. If they are taken out seven at a time, there are no eggs left over. How many eggs are there in the basket? 5

(c) If p is a prime then prove that there exist no positive integers a and b such that $a^2 = pb^2$. 3

(d) Let p be a prime and $n \geq 1$ be any integer.

If $f(x)=a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ is a polynomial of degree n modulo p, then show that the congruence $f(x) \equiv 0 (mod\, p)$ has at most n mutually incongruent solution modulo p. 7

5. Answer **either** [(a) and (b)] **or** [(c) and (d)] :

(a) Show that an odd prime p can be represented as sum of two squares if and only if $p \equiv 1 (mod\, 4)$ 7

(b) If $n \geq 1$ is an integer then show that

$$\prod_{d/n} d = n^{\tau(n)/2}.$$ 3

(c) Find all positive solutions of $x^2+y^2=z^2$ where $0<z<30$. 3

(d) If f and g are two arithmetic functions, then show that the following conditions are equivalent : 7

(i) $f(n) = \sum_{d/n} g(d)$

(ii) $g(n) = \sum_{d/n} \mu(d) f\left(\frac{n}{d}\right) = \sum_{d/n} \mu\left(\frac{n}{d}\right) f(d)$

6. Answer **either** [(a) and (b)] **or** [(c) and (d)] :

(a) Define Boolean Algebra. If A is any finite set, then show that the power set $P(A)$ form a Boolean algebra. Show that there cannot exist a Boolean algebra with three elements. 1+2+2=5

(b) Determine whether the following argument is logically correct or not :

"If I study then I will not fail in discrete mathematics. If I do not play PUBG then I will study. But I failed in discrete mathematics. Therefore, I played PUBG." 5

(c) Find a switching circuit which realizes the Boolean expression : 3

$$x\left(y(z+w)+z(u+v)\right)$$

(d) Show that the collection of connectives $\{\neg, \wedge, \vee\}$ is an adequate system. Hence deduce that $\{\neg, \wedge\}$ form an adequate system of connectives. 5+2=7

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3 (Sem-6) MAT M 5

2020

MATHEMATICS

(Major)

Paper : 6·5

(Graph and Combinatories)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

(a) The value of $2P(n, n-2)$ is

(i) $P(2n, n)$

(ii) $P(n, n-2)$

(iii) $P(n, n)$

(iv) None of these.

Contd.

(b) Find how many functions are there from X to Y where $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$.

(c) The number of vertices of odd degree in a graph is—

(i) always even

(ii) always odd

(iii) can be even as well as odd

(iv) None of above.

(d) The number of vertex in a loop is :

(i) 0

(ii) 1

(iii) 2

(iv) 4

(e) Which of the following statements are true?

(i) Every cycle is a Hamiltonian graph.

(ii) Any graph obtained by adding edges to a Hamiltonian graph is also Hamiltonian.

(iii) A Hamiltonian graph always has a pendent vertex.

(iv) Trees are always Hamiltonian.

(f) Determine True **or** False of the following statement :

"$K_{3,3}$ is non-planar".

(g) Define Eulerian graph.

2. Answer the following questions : 2×4=8

(a) Prove that every graph is an intersection graph.

(b) Represent the graph $G(V, E)$ where the vertex set V and the edge set E are as follows :

$$V = \{1, 2, 3, 4\}$$

$$E = \{(x, y) : x + y \text{ is odd}\}$$

(c) A connected planar graph has nine vertices having degrees 2, 2, 2, 3, 3, 3, 4, 4 and 5. How many edges are there?

(d) Does there exist a tree T with 8 vertices such that, the sum of degree of vertices is 16? Justify your answer.

3. Answer the following questions : 5×3=15

(a) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3 and 7.

(b) There exists no simple graph corresponding to the following degree sequences 2, 2, 4, 4, 2. Justify the above statement.

Or

Show that a complete graph with n vertices consists of $\frac{n(n-1)}{2}$ edges.

(c) Prove that a connected graph is bipartite if and only if it contains no odd cycles.

Or

If a graph G is a tree then prove that every two vertices of G are joined by unique path.

4. Answer ***any one*** part :

(a) Prove that a connected graph G is Eulerian if and only if every vertex of G has even degree. 10

(b) (i) For a graph G, prove that $K(G) \leq \lambda(G) \leq \delta(G)$ The symbols have their usual meaning. 6

(ii) Among all graphs with p vertices and q edges, prove that the maximum connectivity is 0 when $q < p-1$ and $\left[\frac{2q}{p}\right]$ when $q \geq p-1$. 4

5. Answer ***any one*** part : 10

(a) If in a graph G has $n \geq 3$ vertices and every vertices has degree at least $\frac{n}{2}$, then G is Hamiltonian.

(b) Let G be a graph of n vertices. If the sum of the degrees of each pair of vertices in G is $n-1$ or larger, then prove that there exists a Hamiltonian path in G.

6. Answer ***any one*** part :

(a) (i) In how many ways can 21 identical books on English and 19 identical books on Hindi be placed in a row on a shelf, so that two books on Hindi may not be together? 4

(ii) Enumerate the number of non-negative integral to the inequality

$$x_1 + x_2 + x_3 + x_4 + x_5 \le 19$$

6

(b) (i) How many outcomes are possible by casting a 6 faced die 10 times? 4

(ii) How many integers solution are there to the equation

$x_1 + x_2 + x_3 + x_4 = 13, \quad 0 \le x_i \le 5;$

$i = 1, 2, 3, 4$? 6