3 (Sem–3/CBCS) MAT SE 1/2

2023

MATHEMATICS

(Skill Enhancement Course)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–SE–3014

(Computer Algebra Systems and Related Software)

OPTION–B

Paper : MAT–SE–3024

(Combinatorics and Graphy Theory)

Full Marks : 50

Time : Two hours

The figures in the margin indicate full marks for the questions.

OPTION-A

Paper : MAT-SE-3014

(Computer Algebra Systems and Related Software)

1. Answer the following :   1×4=4

    (a) Fill in the blank :

    The ; (semicolon) symbol is used to _____ commands.

    (b) Write the Maple command to calculate π.

    (c) Write *three* well known constants in Mathematica.

    (d) 'Take' command can be used to extract submatrices of a given matrix.
    *(Write True **or** Flase)*

2. Answer the following questions :   2×3=6

    (a) How to use a computer algebra system as a calculator ?

    (b) Why do all Mathematica command names begin with capital letters ?

    (c) What do you mean by span and linear independence ?

3. Answer *any two* :   5×2=10

    (a) Write the rules when defining a function in Mathematica.

    (b) Write the plot commands in Mathematica, Maple and Maxima for the function $y = \sin x$ for $0 \le x \le 2\pi$.

    (c) Let $A = (aij)_{3\times3}$ be a matrix. Write the commands in Mathematica to extract the diagonal elements from the matrix and obtain the adjoint of A. Hence write the commands to find inverse of A, without using the Mathematica command 'Inverse $[A]$'.

    (d) What is Gaussian elimination ? Using Mathematica command to find the reduced row echelon form of the matrix

    $$\begin{pmatrix} 1 & 1 & 4 & 25 \\ 2 & 1 & 0 & 7 \\ -3 & 0 & 1 & -1 \end{pmatrix}$$

4. Answer *any three* questions : $10 \times 3 = 30$

(a) Define a function $f(x) = x^{3x} + sinx$. Write programs for differentiate and integrate of $f(x)$ in Mathematica, Maple and Maxima.

(b) (i) Write short notes on Mathematica and Maple as a calculator.

(ii) Write a short note on built in functions and user-defined functions in Maxima. Give examples.

(c) Mention the plot commands in Mathematica, Maple and Maxima for the following :

(i) $z = \bar{e}^{(x^2 + y^2)}$ for $-2 \le x, y \le 2$

(ii) Three lines $y = 4x + 1$, $y = -x + 4$ and $y = 9x - 8$ for $0 \le x \le 2$

(d) Write a program using Newton's method to find the square root of 81.

(e) Write Mathematica code to solve the non homogeneous system of linear equations $mx = b$

where

$$m = \begin{pmatrix} 1 & 5 & -4 & 1 \\ 3 & 4 & -1 & 2 \\ 3 & 2 & 1 & 5 \\ 0 & -6 & 7 & 1 \end{pmatrix}$$

$$x = \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

(f) What is QR-Decomposition ? Determine QR-Decomposition in the matrix

$$m = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$
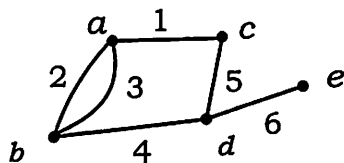
using commands of a suitable computer system.

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OPTION-B

Paper : MAT-SE-3024

(Combinatorics and Graphy Theory)

1. Answer the following : 1×4=4

(a) State true **or** false :

Edges 2 and 3 are adjacent.
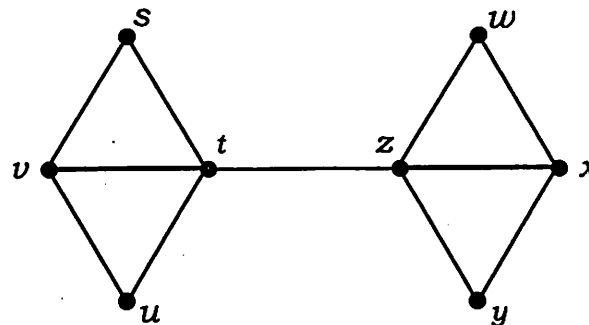
(b) Fill in the blank :

The number of vertices of odd degree in a graph is always _____ .

(c) What is a regular graph ?

(d) What is an order relation on a set ?

2. Answer the following : 2×3=6

(a) Write down all the paths between s and y in the following graph :

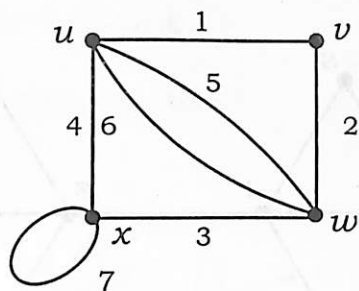(b) Prove that there are no 3-regular graphs with seven vertices.

(c) Prove by bijection that the total number of subsets of a set of order n is 2^n.

3. Answer **any two** parts from the following : 5×2=10

(a) Explain why a tree with n vertices has $n-1$ edges.

(b) For the following graph, write down the following :

(i) a walk of length 7 between u and w

(ii) all the cycles of lengths 1, 2, 3 and 4

(iii) a path of maximum length

(iv) a self loop

(v) a subgraph with 4 vertices and 4 edges.

(c) Check if the following graphs are Eulerian :

(i) K_8, the complete graph.

(ii) $K_{8,8}$, the complete bipartite graph.

(d) Give the combinatorial proof :

Let n and k be positive integers, then

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

4. Explain the 'Four Cubes Problem'. 10

Or

Explain with example blow graph is used to represent interpersonal relationships in social sciences.

5. Define Hamiltonian cycles and Hamiltonian graph. Discuss about Hamiltonian cycles on a dodecahedron. 2+2+6=10

Or

Define Euler graph. Draw *two* graphs such that one is Euler and the other is not. Show that a connected graph is Eulerian if and only if each vertex has even degree.

1+2+7=10

6. In the set of positive integer from 1 to 250, find :

2+4+4=10

(a) How many of these integers are divisible by 3 *or* 5 ?

(b) How many of these integers are divisible by 3 *or* 5 but not by 7 ?

(c) How many of these integers are divisible by 3 *or* 5 *or* 7.

Or

Define lex and colex listing of combinations. In the lex and colex listing of all 3-subsets of [7] find :

2+4+4=10

(a) The 18th subset in the lex order

(b) The rank of the subset {1,4,7} in the colex order

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