Total number of printed pages–7

3 (Sem–3/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA–HC–3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

(a) The standard error of the sampling distribution of the statistic $(\overline{x})$ is *(Choose the correct option)*

(i) $\sigma^2 \sqrt{2/n}$

(ii) $\sigma/\sqrt{n}$

(iii) $\sqrt{\sigma^2/2n}$

(iv) None of the above

Contd.

(b) The cumulative distribution function of the largest order statistic $X_{(n)}$ is given by ________ . *(Fill in the blank)*

(c) For large n if $X \sim N(nP, nPQ)$, then

$$Z = \frac{X - nP}{\sqrt{nPQ}}$$ follows

(Choose the correct option)

(i) $N(0, \sigma^2)$

(ii) $N(\mu, \sigma^2)$

(iii) $N(0, 1)$

(iv) $N(\mu, \sigma)$

(d) If $X_i, i = 1, 2, \ldots n$ are n independent normal variates with mean (μ_i) and SD (σ_i), then $\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma} \right)^2$ is a chi-square variate with n d.f.

(Write true or false)

(e) The probability of type I error is called ________ . *(Fill in the blank)*

(f) State the pdf of Fisher's t-distribution.

(g) Equality of two population variances can be tested by

(Choose the correct option)

(i) t-test

(ii) F-test

(iii) Both (i) and (ii)

(iv) None of the above

2. Answer the following questions : 2×4=8

(a) Derive the cumulative distribution function of $X_{(1)}$.

(b) Write *any two* applications of chi-square statistic.

(c) Explain one tailed and two tailed tests.

(d) Write the assumptions for students t-test.

3. Answer **any three** questions from the following : 5×3=15

(a) Explain in brief the test used for testing the difference between two proportions for large samples.

(b) Find the joint distribution of r^{th} and s^{th} order statistics $(r < s)$ in taking random sample from a continuous distribution.

(c) Derive cumulant generating function (c.g.f.) of chi-square distribution. Also find its mean and variance using c.g.f.

(d) Define F statistic. Write down the p.d.f. of Snedecor's F distribution. Derive the mode of F distribution. 1+1+3=5

(e) In $F(n_1, n_2)$ distribution and if $n_2 \to \infty$, then prove that $n_1 F$ follows chi-square distribution with n_1 d.f.

*Answer **either** 4. (a) **or** 4. (b) :*

4. (a) Obtain the distribution of sample median in case of order statistics. 10

(b) (i) Let X_1 and X_2 be two independent normal variates with the same normal distribution $N(\mu, \sigma^2)$. Obtain the distribution of

$$Y = \frac{X_1 + X_2 - 2\mu}{\sqrt{|X_1 - X_2|^2}}.$$ 5

(ii) If X is t-distributed with K degrees of freedom, show that $\frac{1}{1+(X^2/K)}$ has a beta distribution. 5

Answer ***either*** 5. *(a)* ***or*** 5. *(b)* :

5. *(a)* If X_1 and X_2 are two independent chi-square variate with n_1 and n_2 d.f. respectively, then show that X_1/X_2 is a $\beta_2(n_1/2, n_2/2)$ variate. 10

(b) *(i)* Describe the steps in detail for testing a statistical hypothesis. 5

(ii) For *t*-distribution with *n* d.f., derive the mean deviation about mean. 5

Answer ***either*** 6. *(a)* ***or*** 6. *(b)* :

6. *(a)* *(i)* Derive the probability density function of student's *t*. 7

(ii) Comment on the graph of *t*-distribution. 3

(b) Write three applications of *F* distribution.

Let X_1 and X_2 be a random sample of size 2 from $N(0, 1)$ and Y_1 and Y_2 be a random sample of size 2 from $N(1, 1)$ and let Y_i's be independent of X_i's. Find the distribution of the following : 3+7=10

(i) $\dfrac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$

(ii) $\dfrac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$

1000

Total number of printed pages–7

3 (Sem–3/CBCS) STA HC 2

2023

STATISTICS

(Honours Core)

Paper : STA–HC–3026

(Survey Sampling and Indian Official Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

   (a) The number of possible samples of size n out of N population size in SRSWOR is equal to

   (i) $^{N}C_{n}$

   (ii) N^{n}

Contd.

(iii) $\frac{(N-n)}{N}$

(iv) n/N

(Choose the correct answer)

(b) A selection procedure of sampling having no involvement of probability is known as ______. *(Fill in the blank)*

(c) Sub sampling is also known as two stage sampling. *(True or False)*

(d) The sampling procedure where the probability of selection is proportional to the size of the unit is known as

(i) simple random sampling with replacement

(ii) probability proportional to size sampling

(iii) stratified sampling

(iv) None of the above

(Choose the correct option)

(e) A complete list of units which represents the population to be covered is called the ______. *(Fill in the blank)*

(f) Inverse of sampling fraction is called ______ factor. *(Fill in the blank)*

(g) State the condition under which the regression estimator reduces to the ratio estimator.

2. Answer the following questions briefly : 2×4=8

(a) Name the *three* principles of sampling theory.

(b) Define accuracy and precision.

(c) In what situations the P.P.S sampling is prefered over simple random sampling.

(d) A population of eight households, say *a*, *b*, *c*, *d*, *e*, *f*, *g* and *h*, write down all possible samples of size 3 according to the technique of circular systematic sample.

3. Answer ***any three*** from the following questions : 5×3=15

(a) Prove that in stratified random sampling, the $\overline{y}_{st}$ is an unbiased estimate of population mean. Also find its variance.

(b) Explain the concept of linear and circular systematic sampling.

(c) Explain the cumulative total methods and the Lahiri's method of selecting a probability proportional to size (PPS) sample with replacement.

(d) What are the different sources of errors in a sample survey ? How can these errors be controlled ?

(e) Write a note on origin and function of central statistical organisation (CSO) and its publications.

4. Answer ***either*** *(a)* ***or*** *(b)* of the following questions :

(a) In a stratified random sampling with cost function $C = a + \sum_{i=1}^{k} n_i C_i$ where the overhead cost a is a constant and C_i is the average cost of sampling one unit in the ith stratum.

Prove that $n_i = \dfrac{nN_i S_i / \sqrt{C_i}}{\sum_{i=1}^{k} \left(N_i S_i / \sqrt{C_i}\right)}$

From the above relation state the condition under which a larger sample needs to be taken. 7+3=10

(b) Discuss regression method of estimation. Show that simple regression estimate is a biased estimate of population mean $\overline{Y}_N$. Obtain the variance of the simple regression estimate. 10

5. Answer ***either*** *(a)* ***or*** *(b)* :

(a) Show that in a simple random sampling without replacement of n clusters containing M elements from a population of N clusters, the sample mean $\overline{y}_n$ is an unbiased estimator of $\overline{Y}$ and its varience is given by

$$V(\overline{y}_n) \cong \frac{(1-f)}{nM} S^2 [1 + (M-1)e] \text{ for large}$$

N where ρ is the intracluster correlation co-efficient. 3+7=10

(b) Find an unbiased estimate of the population mean in systematic sampling.

If the population consists of a linear trend of the form

$Y_i = a + b_i$, $i = 1, 2, \ldots\ldots, N$, $N = nk$

then prove that

$V(\overline{y}_{st}) \leq V(\overline{y}_{sys}) \leq V(\overline{y}_n)_R$

(symbols have their usual meanings) 2+8=10

6. Answer ***either*** *(a)* ***or*** *(b)* :

(a) Describe the methods of collection of official statistics in India. In this context discuss the role of Ministry of Statistics and program implementation. 6+4=10

(b) Explain the principal steps involved in the planning and execution of a sample survey. 10

1000

3 (Sem–3/CBCS) STA HC 3

2023

STATISTICS

(Honours Core)

Paper : STA–HC–3036

(Mathematical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : 1×7=7

(a) The least upper bound of the set $\left\{\frac{1}{n}, n \in N\right\}$ is

(i) 1

(ii) 0

(iii) –1

(iv) None of the above

(Pick up the correct option)

(b) Identify the wrong statement :

(i) The intersection of two open sets is open.

(ii) Every open set is an union of open intervals.

(iii) The union of two open sets is closed.

(iv) The set of all integers is countable.

(c) State Bolzano-Weierstrass theorem.

(d) A sequence cannot converge to more than one limit. *(State True or False)*

(e) The value of $\Delta^4 (1-x)^4$, the interval of differencing being unity is

(i) 0

(ii) 1

(iii) 4

(iv) 24

(Choose the correct option)

(f) Given the following data :

Income per day not exceeding (Rs.) :	10	18	20	28	40
Workers :	12	32	68	80	100

To interpolate number of workers for income not exceeding Rs.30 per day, the suitable method is :

(i) Newton's backward formula

(ii) Lagrange's formula

(iii) Binomial expansion method

(iv) Gauss backward formula

(Choose the correct option)

(g) If the n^{th} differences of a tabulated function $f(x)$ are constant, the value of independent variables are taken at equal intervals, then

(i) $f(x)$ is a polynomial of degree n

(ii) $f(x)$ is constant

(iii) $f(x)$ is zero

(iv) $f(x)$ is a polynomial of degree $(n-1)$

(Choose the correct option)

2. Answer the following questions : 2×4=8

(a) Using Lagrange's mean value theorem, prove that

$$\left|\tan^{-1}x-\tan^{-1}y\right|\le|x-y|\ \forall\ x,y\in R$$

(b) Prove that every convergent sequence is bounded.

(c) Show that for any real number x,

$$\lim\frac{x^n}{n!}=0.$$

(d) State Taylor's theorem with Lagrange's and Cauchy's form of remainder.

3. Answer ***any three*** of the following questions : 5×3=15

(a) State and prove Cauchy's first theorem on limits.

(b) Expand $\sin x$ by Maclaurin's infinite series.

(c) State and prove Rolle's theorem.

(d) If four equidistant values u_{-1}, u_0, u_1 and u_2 are given and a value u_x is interpolated by Lagrange's formula, show that

$$u_x=yu_0+xu_1+\frac{y(y^2-1)}{3!}\Delta^2u_{-1}+\frac{x(x^2-1)}{3!}\Delta^2u_0$$

where $x+y=1$.

(e) Show that the n^{th} order divided difference of a polynomial of n^{th} degree is constant.

4. Answer *(a)* **or** *(b)* of the following questions :

(a) (i) Evaluate $\lim\limits_{x\to 0}\frac{x-\tan x}{x^3}$ 3

(ii) Expand $(1+x)^n$ by Maclaurin's infinite series. 7

(b) (i) Prove that a function which is uniformly continuous on an interval is continuous on that interval. 4

(ii) Let $f(x)=\begin{cases} x^P \sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

Obtain p such that (i) $f(x)$ is continuous at $x=0$ (ii) $f(x)$ is differentiable at $x=0$. 6

5. Answer (a) **or** (b) of the following questions :

(a) State and prove Cauchy's general principle of convergence. 10

(b) (i) Solve the difference equation :

$$u_{x+2} - 4u_x = 9x^2$$ 4

(ii) Write a note on use of various interpolation formulae. 6

6. Answer (a) **or** (b):

(a) (i) State Cauchy's n^{th} root test and Leibnitz's test for the convergence of alternating series. 4

(ii) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\dots\dots+\frac{1}{\sqrt{n^2+n}}\right]=1$$

6

(b) (i) Derive Gauss's interpolation formula for central differences. 5

(ii) State and prove Weddle's rule for numerical integration. 5

1000