2015

PHYSICS

(Major)

Paper : 5.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(**Mathematical Methods**)

(*Marks* : 30)

1\. Answer the following questions : 1×4=4

(a) For the complex number $z = 3 - 4i$, find z^4, given that

$$\tan^{-1}\frac{4}{3} = 53 \cdot 13^\circ$$

(b) What does the equation $|z - i| = 2$ represent?

(c) Plot the number $e^{(1-\pi/6^i)}$.

(d) Find the principal value of i^i.

(*Turn Over*)

2. (a) Solve the equation $z^4 + 16 = 0$ and plot the values of z. 2

(b) Prove : $\sin^2 z + \cos^2 z = 1$. 2

3. (a) Check the analyticity of the function $f(z) = \ln z$ and hence find its derivative. 4

(b) Find the principal value of $(2+i)^{1-i}$. 4

Or

Using Cauchy's integral formula, evaluate the integral

$$\oint \frac{z-1}{z^2+1} dz$$

around the contours—

(i) $|z-i|=1$

(ii) $|z|=2$ 2+2=4

4. (a) State and prove Cauchy's integral theorem. 4

(b) Define the following with diagram : 3

(i) Simply connected region

(ii) Multiply connected region

(iii) Equivalent contour

Or

(a) State and prove Taylor's theorem. 4

(b) Find Taylor series expansion about the origin for $\sin \pi z$. 3

5. (a) Define pole and residue. 1

(b) If a function $f(z)$ has an m th order pole at $z=a$, then show that the residue at that singular point is

$$a_{-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \left\{(z-a)^m f(z)\right\}_{z=a}$$

and hence find the residue of

$$f(z) = \frac{e^z}{(z-i)^2}$$

at its pole. 4+2=6

Or

Evaluate the integrals : 3+4=7

(i) $\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^2}$

(ii) $\int_0^{2\pi} \frac{\sin\theta \, d\theta}{1+\cos\theta}$

GROUP—B

(Classical Mechanics)

(*Marks* : 30)

6. Answer the following questions : 1×4=4

(a) What is areal velocity of a particle?

(b) The equation of constraint for a particle moving on or out of the surface of a sphere of radius r is given by

$$x^2 + y^2 + z^2 \geq r^2$$

What are the two types of constraints that can be associated with the motion of the particle?

(c) Write down the expression for the Lagrangian of a free particle in cylindrical polar coordinates.

(d) What is the physical significance of the Hamiltonian of a particle?

7. *(a)* What are cyclic or ignorable coordinates? If a system undergoes translatory motion along a cyclic generalized coordinate q_k, will the Lagrangian of the system be affected? 2

(b) Show that the Poisson bracket of a function with itself is identically zero, i.e., $[u, u] = [v, v] = 0$ where u and v are any two arbitrary functions. 2

Or

Obtain the Lagrangian equation of motion if the Lagrangian has the form $L = -(1 - \dot{q}_j^2)^{1/2}$. Show that the generalized conjugate momentum p_j is conserved. 4

8. Answer any *three* of the following questions : 4×3=12

(a) For a particle subjected to a central force, prove that *(i)* the angular momentum of the particle is a constant of motion, *(ii)* the particle moves in a fixed plane, and *(iii)* the areal velocity of the radius vector remains constant.

(b) The motion of a particle under the influence of a central force is described by $r = a\sin\theta$. Find an expression for the force.

(c) State the d'Alembert's principle. Deduce the Lagrange's equation of motion for a conservative holonomic system using this principle.

(d) The point of suspension of a pendulum moves in the vertically downward direction with constant acceleration a. Find the Lagrangian and hence the equation of motion. What will be its period if the downward acceleration a is the same as that due to gravity?

(e) Show that the Hamiltonian H of a system can be written as

$$H = \sum_j p_j \dot{q}_j - L(q_j, \dot{q}_j, t)$$

where $L(q_j, \dot{q}_j, t)$ is the Lagrangian of the system and p_j are the generalized momenta, q_j are the generalized coordinates and $\dot{q}_j$ are the generalized velocity coordinates.

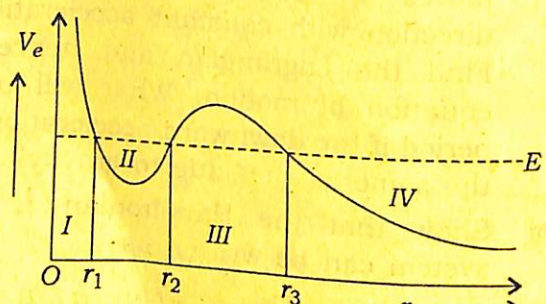
9. Answer any *two* questions : 5×2=10

(a) Assuming attractive inverse square law of force $F(r) = -k/r^2$, where $k > 0$, show that the speed v of the particle in the above field is given by

$$v = \sqrt{\frac{k}{m}\left(\frac{2}{r} \mp \frac{1}{a}\right)}$$

where a is the semi-major axis of the conical path.

(b) Referring to the figure given below, consider an arbitrary potential field caused by a central force. Let us suppose that the total energy E of the particle is represented by the dotted line :



Describe the nature of motion of the particle entering the potential field with energy E in the regions *I*, *II*, *III* and *IV* as shown in the figure. What are turning points of motion?

(Continued)

(c) Using Lagrangian formulation, deduce the equation of motion of a compound pendulum and determine its time period. What is the condition under which the motion of the compound pendulum becomes a simple harmonic motion?

(d) What are the Hamilton's canonical equations of motion? Using Hamilton's canonical equations, derive the equation of motion of a particle moving in a force field in which the potential is given by $v = -kx$, where k is positive.

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2015

PHYSICS

(Major)

Paper : 5.2

(Atomic Physics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×7=7

(a) In Compton scattering as the scattering angle of photon varies from 0 to π, the angle of recoil electron varies from

(i) π to 0

(ii) 0 to π

(iii) $\frac{\pi}{2}$ to 0

(iv) 0 to $\frac{\pi}{2}$

(b) A beam of electromagnetic radiation is allowed to pass through hydrogen gas at room temperature. The radiation has wavelengths extending from UV to

(Turn Over)

IR region. The absorption lines will be observed in the

(i) Lyman series

(ii) Balmer series

(iii) Paschen series

(iv) All of the above

(c) X-rays from a tube pass through a metal foil. The transmitted intensity is I_0. When the thickness of the foil is doubled, the transmitted intensity becomes I. Then

(i) $I_0 = 2I$

(ii) $I_0 = I$

(iii) $I_0 = \frac{I}{2}$

(iv) None of the above

(d) When a magnetic field is applied to an orbital electron, the electron gains ε energy. If the field direction is reversed, the change in energy will be

(i) 2ε

(ii) 0

(iii) ε

(iv) $\frac{\varepsilon}{2}$

(e) Which among the following is *not* true about Raman effect?

(i) Scattered lines have frequencies greater and lesser than the incident frequency

(ii) Frequency shift is independent of the scatterer

(iii) Elastic collision takes place between photon and molecule

(iv) Scattered lines are polarized

(f) The atoms taken in the Stern and Gerlach experiment had

(i) $J = L + S$

(ii) $J = L$

(iii) $J = S$

(iv) $J = 0$

(g) If λ_1 and λ_2 are the wavelengths of the K_β lines in the characteristic X-rays of two elements with atomic numbers z_1 and z_2, then

(i) $\lambda_1 < \lambda_2$ if $z_1 > z_2$

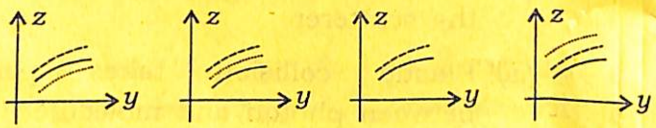
(ii) $\lambda_1 = \lambda_2$ if $z_1 > z_2$

(iii) $\lambda_1 > \lambda_2$ if $z_1 > z_2$

(iv) $2\lambda_1 = \lambda_2$ if $2z_1 < z_2$

2. Answer any *four* questions from the following : 2×4=8

(a) When four different samples of the same element are taken in Thomson's experiment, the following four sets of parabolic traces are obtained :



What inference can be drawn as regards the isotopic composition of the element from the traces?

(b) The velocity selector in a Bainbridge mass spectrograph uses an electric field of 30 kV m^{-1} and a magnetic field of 2×10^3 gauss. Find the speed of the ions which will ultimately strike the photographic plate.

(c) An atom has 18 electrons in *M*-shell. Write symbolically how the electrons are distributed in the *s*, *p*, *d*, ··· sub-shells, using proper spectral notation.

(d) Show that the unit of Bohr magneton may also be written as $\text{A}\,\text{m}^2$.

(e) The energy levels of an atom are respectively −15 eV, −9 eV, −6 eV and −3 eV. What is the maximum momentum of photon that the atom can emit?

(f) Two fields are applied one after the other on an electron confined in an orbit. In one case the orbit is distorted, and in the other case the electron speeds up. Which field corresponds to which case? Explain with the help of two diagrams.

3. Answer question *(a)* and any *two* from *(b)*, *(c)* and *(d)* : 5×3=15

(a) A hydrogen atom in its ground state undergoes a collision and turns into an ion. Find the wavelength of light emitted if it regains its ground state.

If the electron is replaced by a negative muon (μ^-) with mass $m_\mu \approx 200\, m_e$, then calculate the wavelength of light emitted by the modified atom in the transition $n = 3$ to $n = 2$. Assume Rydberg constant $R = 1 \times 10^7$ m^{-1}.

In which region of the electromagnetic spectrum this radiation will fall?

(b) What is the difference between anomalous Zeeman effect and Paschen-Back effect? A proton is shot into a tin nucleus ($z = 50$). If the proton stops at a distance of $1 \cdot 6 \times 10^{-3}$ Å from the centre of the nucleus before retracing its path, then find the energy (in MeV) with which the proton was shot.

(c) What is Ritz combination principle? Using this principle, show that the wave number of the second line in Balmer series is the sum of the wave numbers of the first lines in Paschen series and Balmer series.

(d) Write a short note on any *one* of the following :

(i) Alkali spectra

(ii) Normal Zeeman effect

(iii) Sommerfeld modification of Bohr's atom model

4. Answer questions (a) and (b); and any *one* from (c), (d) and (e) :

(a) Draw a neat labelled diagram of Aston's mass spectrograph.

Discuss mathematically how the electric field disperses and the magnetic field converges a beam of ions in the apparatus. Hence, arrive at the condition of focussing of the beam. 3+6+1=10

(Continued)

(b) Deduce Rutherford's formula on scattering of α-particles from a metal target. 10

(c) An X-ray photon of frequency ν strikes a free electron of rest mass m_0. The photon undergoes an inelastic collision and gets scattered at an angle θ and has a reduced frequency ν′. Show that

$$\nu' = \frac{\nu}{1 + \frac{2h\nu}{m_0 c^2} \sin^2 \frac{\theta}{2}}$$ 10

(d) Give the mathematical theory of Stern-Gerlach experiment. Draw the schematic diagram of the apparatus.

In the apparatus the magnetic field has a gradient of 2 Wb m^{-2} per mm and a neutral atom of mass $9 \cdot 27 \times 10^{-22}$ g is projected at a speed of 400 ms^{-1}. The atom travels a distance of 40 cm inside the field and is deflected from its original path by an amount of 2 cm, then find the magnetic moment of the atom in the unit of Bohr magneton.

5+1+4=10

(e) Give the quantum theory of Raman effect. What are Stokes' and anti-Stokes' lines? In an experiment on Thomson's determination of q / M of positive rays an

(Turn Over)

x-y-z coordinate system is such that the origin of the system is at the point at which the ion enters the fields $\vec{E}$ and $\vec{B}$ where $\vec{E} = \hat{k}E$ and $\vec{B} = \hat{k}B$, and the velocity of the ion of mass M and charge q is $\vec{v} = \hat{j}v$. If the fields extend to a length l, then determine the final coordinates $P(x, y, z)$ of the ion just after it comes out of the field region.

4+½+½+5=10

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2015

PHYSICS

(Major)

Paper : 5.3

(Quantum Mechanics and Astrophysics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

Write the answers to the two Groups in separate books

GROUP—A

(Quantum Mechanics)

(*Marks* : 40)

1. Answer any *four* questions as directed : 1×4=4

(a) What physical phenomenon proves the particle nature of light?

(b) A krypton atom emits a photon of orange light with wavelength $\lambda \approx 606$ nm. What is the corresponding photon energy?

(c) In Compton scattering, which one of the following is true?

(i) The wavelength of scattered light is same as the wavelength of incident light

(ii) The incident photon is absorbed by the electron

(iii) The scattered wavelength is larger than the incident wavelength

(iv) The scattered wavelength is smaller than the incident wavelength

(Choose the correct option)

(d) According to de Broglie hypothesis, the kinetic energy of a particle of mass m is

(i) $k = h^2 / 2m\lambda^2$

(ii) $k = mc^2$

(iii) $k = mc^2\left(1 - \frac{v^2}{c^2}\right)$

(iv) $k = \frac{mc^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$

(Choose the correct option)

(Continued)

(e) Can the particle and wave nature be simultaneously observed?

(f) An electron is confined within a region of width $1 \cdot 0 \times 10^{-10}$ m. Estimate the minimum uncertainty in the momentum of the particle.
(Given $\hbar \approx 1 \cdot 05 \times 10^{-34}$ J-sec)

2. Answer any *two* questions : 3×2=6

(a) A non-relativistic free particle of mass m has kinetic energy k. Obtain an expression for the de Broglie wavelength. What is the de Broglie wavelength for an electron having kinetic energy 800 eV? Given electron mass $m_e \approx 9 \cdot 1 \times 10^{-31}$ kg. 1½+1½=3

(b) A particle with total energy E is influenced by a potential energy $V(x)$. Show that the one-dimensional Schrödinger equation can be written in the form

$$\left[\frac{d^2}{dx^2} + k^2 - U(x)\right]\psi(x) = 0$$

(Turn Over)

where

$$k^2 = 2mE/\hbar^2 \text{ and}$$
$$U(x) = 2mV(x)/\hbar^2$$

If the wave function of a particle is $\psi(x, y, z, t) = \psi(x, y, z)e^{-iEt/\hbar}$, then show that the probability of finding the particle at the point (x, y, z) is independent of time. 2+1=3

(c) If the wave function of a particle confined in a box of length L is

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$$

obtain the expectation value of the position $\langle x \rangle$ of the particle. 3

(d) Two quantum particles are travelling along X-axis in opposite direction. Their wave functions are combined to produce a resultant wave $\psi(x, t) = A\cos(kx)e^{-i\omega t}$. What is the probability of current density j for this wave function? Interpret your result. 2+1=3

(Continued)

3. Answer any *two* questions : 5×2=10

(a) The photoelectric work function W for lithium is 2·3 eV. Find the threshold frequency. If the ultraviolet light of wavelength $\lambda = 3000$ Å is incident on a lithium surface, calculate the maximum kinetic energy of the photoelectrons. Briefly discuss how the wave theory of light fails to explain the photoelectric effect. 1+2+2=5

(b) Prove that the angle ϕ, at which the electron in Compton effect recoils, is related to the angle θ of scattered photon as

$$\tan\phi = \frac{\cot(\theta/2)}{1 + E_0/mc^2}$$

Where $E_0 = hc/\lambda =$ energy of incident photon and m is the rest mass of the electron.

An X-ray photon with wavelength $\lambda = 1$ Å is scattered by a free electron at rest. The scattering angle is $\theta = 60°$ from the incident direction. Calculate the Compton shift $\Delta\lambda$. 3+2=5

(Turn Over)

(c) In an electron diffraction experiment, electrons are accelerated by an electric potential V. Show that the de Broglie wavelength of an electron of mass m is

$$\lambda = \frac{h}{\sqrt{2meV}}$$

where e is the electronic charge. The kinetic energy of a particle in a gas with temperature T (kelvin) is $\frac{3}{2}k_B T$. What is the de Broglie wavelength of a thermal neutron at $T = 300$ K? Here k_B is the Boltzmann constant $\approx 1 \cdot 38 \times 10^{-23}$ J/K and neutron mass is $m \approx 1 \cdot 6 \times 10^{-27}$ kg. Write down the relativistic formula for de Broglie wavelength. 2+2+1=5

4. Answer any *two* questions : 5×2=10

(a) What are the properties satisfied by a physical wave function? Normalize the wave function

$$\psi(x) = Ae^{-(\frac{\alpha}{2})x^2}$$

to unity in the domain $x \in [-\infty, \infty]$. Here α is a constant. Given $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. 2+3=5

(Continued)

(b) A particle of mass m is confined in a one-dimensional potential box of infinite height given as

$$V(x) = \infty \text{ for } x = 0, a$$
$$= 0 \text{ for } 0 < x < a$$

Show that the energy of the particle in nth quantum state is

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$ 5

(c) For a linear harmonic oscillator potential $V(x) = \frac{k}{2}x^2$, show that the Schrödinger equation (time-independent) takes the form

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \xi^2)\psi(\xi) = 0$$

where

$$\lambda = 2E / \hbar\omega, \ \omega = \sqrt{k / m}$$

and $\xi^2 = \left(\frac{mk}{\hbar^2}\right)^{1/2} x^2 = \left(\frac{m\omega}{\hbar}\right) x^2$

or $\xi = \alpha x$ where $\alpha = \sqrt{m\omega / \hbar}$. 5

(Turn Over)

5. Answer any *two* questions : 5×2=10

(a) Normalize the wave function $\psi(x) = A/x^2$ between $x = 1$, $x = 3$. What is the probability of finding a particle between $x = 4$ and $x = 5$? 2+3=5

(b) What is the significance of Heisenberg's uncertainty principle? A proton is confined to a nucleus of dimension $\Delta x \approx 10^{-15}$ m. Calculate the uncertainty in its momentum. What is the minimum kinetic energy of the proton? Given proton mass $m \approx 1 \cdot 6 \times 10^{-27}$ kg. 1+2+2=5

(c) Briefly discuss the Davisson-Germer experiment and its implications. 5

GROUP—B

(Astrophysics)

(*Marks* : 20)

6. Answer any *three* from the following : 2×3=6

(a) Name two bright stars in the night sky. Show the right ascension and declination in a neat celestial diagram. 1+1=2

(b) Are the altitude and azimuth of a star same for all observers on the earth? What do you mean by ecliptic? 1+1=2

(c) The star ξ Ursae Majoris has a parallax angle of $\theta \approx 0'' \cdot 127$. Calculate its distance. 2

(d) An astronomer wants to observe a star with right ascension (α) and declination (δ) as

$$(23^h 20^m 39^s + 18°08'33'')$$

Which hemisphere of the earth would be the best for observing? If the star is at the meridian, what would be the sidereal time? 1+1=2

(e) A star has apparent blue magnitude $m_B = 12 \cdot 4$ and the colour index is $m_B - m_V = 0 \cdot 6$. If the absolute magnitude in visual band is $M_V = 6 \cdot 8$, calculate the distance to the star. 2

7. Answer any *two* of the following : 4×2=8

(a) Draw a neat H-R diagram showing the main sequence, red giant, red supergiant and the white dwarf stars. Identify the location of a hot blue and a reddish cool star in the main sequence. Show the evolutionary track of a sun-like star in the H-R diagram. 1½+1+1½=4

(b) What is the energy generation mechanism inside the main-sequence stars? Which reaction cycle dominates the energy production in massive stars with high central temperature? Discuss the proton-proton chain for the synthesis of helium. 1+1+2=4

(c) The peak wavelength emitted by a star is $\lambda_{max} \approx 4000$ Å. Calculate the surface temperature of the star. Given Wien's constant, $b \approx 0 \cdot 29$ cm-K. The luminosity L, radius R and surface temperature T of a star are related by Stefan-Boltzmann law $L = 4\pi R^2 \sigma T^4$. The corresponding quantities for the sun are $L_\odot$, $R_\odot$ and $T_\odot$.

(Continued)

A star's surface temperature is $\frac{1}{2} T_\odot$ but its luminosity is $10^4 L_\odot$. How much big is the star compared to the sun? Can you give example of one such star in the night sky? 1½+1½+1=4

8. Write short notes on any *two* of the following : 3×2=6

(a) Red giants

(b) Astronomical coordinate system

(c) Supernova

(d) Expanding universe

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