Total number of printed pages–7

3 (Sem–5) MAT M 1

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·1

(Real and Complex Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

   (a) Define limit of a function of two variables.

   (b) Evaluate :

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+\left(x^2-y^2\right)^2}$$

Contd.

(c) Give an example of a function which is not continuous but Riemann integrable.

(d) When is an improper integral said to be convergent ?

(e) Find the fixed points of the transformation $w = \frac{2z+3}{z-4}$, z is a complex number.

(f) Let C_1 and C_2 be two simple closed curves, then $\oint_{C_1} z dz = \oint_{C_2} z dz$.

(State true or false)

(g) Verify whether the transformation $w = z^3$ is conformal or not at all points of the region $|z| < 1$.

2. Answer the following questions : 2×4=8

(a) Show that the following function is discontinuous at the origin :

$$f(x, y) = \begin{cases} \frac{1}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

(b) Show that a constant function k is integrable and

$$\int_a^b k\,dx = k(b-a)$$

(c) Test the convergence of

$$\int_0^{\pi/2} \frac{\sin x}{x^p}\,dx$$

(d) Evaluate : $\lim_{z \to i} \frac{z^{10}+1}{z^6+1}$

3. Answer ***any three*** parts : 5×3=15

(a) Show that $f(xy, z-2x)=0$ satisfies, under suitable conditions, the equation $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2x$. What are these conditions ?

(b) Show that $\int_1^2 f\,dx = \frac{11}{2}$, where $f(x) = 3x+1$.

(c) Show that the integral

$$\int_0^{\pi/2} \frac{\sin^m x}{x^n}\, dx$$

exits, iff $n < m + 1$.

(d) Prove that the real and imaginary parts of an analytic function of a complex variable when expressed in polar form satisfy the equation

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0$$

(e) Evaluate :

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\, dz$$, where C is the circle $|z| = 3$.

4. Answer ***any three*** parts : 10×3=30

(a) (i) Show that the function f, where

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , \text{ if } x^2 + y^2 \neq 0 \\ 0 & , \text{ if } x = y = 0 \end{cases}$$

is continuous, possesses partial derivative, but is not differentiable at the origin. 6

(ii) Show that the function

$$f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & , (x,y)\neq(0,0)\\ 0 & , (x,y)=(0,0)\end{cases}$$

is continuous at the origin. 4

(b) (i) Prove that $\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}$ is invariant for change of rectangular axes. 6

(ii) Expand $x^2y+3y-2$ in powers of $x-1$ and $y+2$. 4

(c) (i) If $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$, then find the maximum value of xyz. 5

(ii) Prove that the improper integral $\int_a^b f dx$ converges if and only if to every $\varepsilon>0$, there corresponds $\delta>0$ such that

$$\left|\int_{a+\lambda_1}^{a+\lambda_2} f dx\right|<\varepsilon,\quad 0<\lambda_1,\lambda_2<\delta.$$ 5

Contd.

(d) (i) The oscillation of a bounded function f on an interval $[a, b]$ is the supremum of the sit $\{|f(x_1) - f(x_2)| : x_1, x_2 \in [a, b]\}$ of numbers. 5

(ii) Show that the function $[x]$, where $[x]$ denotes the greatest integer not greater than x, is integrable in $[0, 3]$ and $\int_0^3 [x]\,dx = 3$. 5

(e) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. Hence find v such that $f(z) = u + iv$ is analytic. 4+6=10

(f) (i) Evaluate :

$$\oint_C \frac{z^2}{(z-1)(z-2)} dz$$

and

$$\oint_C \frac{e^{2z}}{(z+1)^4} dz$$, where C is the circle $|z| = 3$. 3+2=5

(ii) Determine the region of the w plane into which each of the following is mapped by the transformation $w = z^2$: 2+3=5

(A) First quadrant of the z-plane

(B) Region bounded by $x = 1$, $y = 1$ and $x + y = 1$

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3 (Sem-5) MAT M2

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·2

(Topology)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1\times7=7$

(a) Describe the closed sphere $S[-1, 1]$ for the usual metric on $\mathbb{R}$.

(b) Find the closure of the set $A=\left\{1, \frac{1}{2}, \frac{1}{3}, ...\right\}$ in the real line $\mathbb{R}$ with the usual metric.

Contd.

(c) A subset of a metric space (X, d) is open if and only if

(i) $A = A^0$

(ii) $A \neq A^0$

(iii) $A = \overline{A}$

(iv) $A \neq \overline{A}$ (Choose the correct one)

(d) Define a complete metric space.

(e) Let (X, I) be an indiscrete topological space. What is the derived set of any subset A of X?

(f) Consider the topological space (X, T) where $X = \{a, b, c, d\}$ and $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$.

A mapping $f : X \to X$ is defined as

$f(a) = b,\ f(b) = d,\ f(c) = b,\ f(d) = e.$

State whether f is continuous at c or not?

(g) What do you mean by a Banach space?

2. Answer the following questions : 2×4=8

(a) Show that every subset of a discrete metric space is closed.

(b) Prove that every convergent sequence in a metric space is a Cauchy sequence.

(c) Let X be a set and $T = \{\phi, A, B, X\}$ where A and B are non-empty distinct proper subsets of X. Find what conditions A and B must satisfy in order that T may be a topology for X.

(d) Prove that every normed linear space is a metric space.

3. Answer the following questions : 5×3=15

(a) (i) Let (X, d) be a metric space and

$$d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \ \forall x, y \in X.$$

Prove that d' is a metric on X. 3

(ii) Is intersection of an arbitrary family of open sets is open in any metric space? Justify it. 2

(b) (i) Let $X = \{a, b, c, d, e\}$ and $A = \{\{a, b, c\}, \{c, d\}, \{d, e\}\}$.

Find the topology on X generated by A. 2

(ii) Let $f : X \to Y$ be a function from a non-empty set X into a topological space (Y, U). If $T = \{f^{-1}(G) | G \in U\}$, then show that T is a topology on X. 3

Or

Prove that in a topological space (X, T), if $A \subseteq X$, then $\overline{A} = A \cup D(A)$. 5

(c) Show that $\mathbb{R}^n$ is a normed linear space with some suitable norm. 5

Or

Let $(X, <\cdot>)$ be an inner product space. Prove that for all $x, y \in X$,

$$4 < x, y >= \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$$

5

4. Answer the following questions : 10×3=30

(a) (i) Let (X, d) be a metric space. Prove that a subset F of X is closed if and only if its complement F^C in X is open. 5

(ii) Let (X, d) be a metric space and $Y \subseteq X$. Show that a subset A of Y is open in (Y, dy) if and only if there exists a set G open in (X, d) such that $A = G \cap Y$. 5

Or

Let (X, d) be a metric space and $\langle F_n \rangle$ be a nested sequence of non-empty closed subsets of X such that $d(F_n) \to 0$ as $n \to \infty$. Prove that X is complete if and only if $\bigcap_{n=1}^{\infty} F_n$ consists of exactly one point. 10

(b) (i) Let (X, d_1) and (Y, d_2) be two metric spaces. Prove that the mapping $f : X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y. 6

(ii) If (X, d_1) and (Y, d_2) be two metric spaces, then prove that the mapping $f : X \to y$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$, for every subset A of X. 4

Or

Let (X, d_1) be a metric space and (Y, d_2) be a complete metric space. If f is a uniformly continuous mapping from a dense subset of X into Y, then prove that f can be extended uniquely to a uniformly continuous mapping $g : X \to Y$. 10

(c) Define a connected metric space. Prove that a non-empty subset of $\mathbb{R}$ (with usual metric) is connected if and only if it is an interval. 1+9=10

Or

Prove that a subset A of the real line $\mathbb{R}$ is compact if and only if it is closed and bounded. 10

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (i) Write the condition under which one may have an infinite number of great circles through two given points.

   (ii) Define polar triangle and its primitive.

   (iii) State the third law of Kepler.

   (iv) What do you mean by circumpolar star ?

Contd.

(v) Define primary circle.

(vi) What is parallactic ellipse?

(vii) What is the declination of the pole of the ecliptic?

2. Answer the following questions: 2×4=8

(a) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(b) Discuss the effect of refraction on sunrise.

(c) Prove that section of a sphere by a plane is a circle.

(d) Show that right ascension α and declination δ of the sun is always connected by the equation

$$\tan \delta = \tan \varepsilon \sin \alpha$$

ε being obliquity of the ecliptic.

3. Answer ***any three*** questions of the following: 5×3=15

(a) Deduce Kepler's laws from Newton's law of gravitation.

(b) If H be the hour angle of a star of declination δ when its azimuth is A and H' when the azimuth is $(180° + A)$, show that

$$\tan\phi = \frac{\cos\frac{1}{2}(H'+H)}{\cos\frac{1}{2}(H'-H)}$$

(c) In an equilateral spherical triangle ABC, prove that $2\cos\frac{a}{2}\sin\frac{A}{2} = 1$.

(d) Define geocentric parallax. Show that geocentric parallax of a heavenly body varies as the sine of its apparent zenith distance.

(e) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi\sec\delta$, where the symbols have their usual meanings.

4. Answer ***any three*** questions of the following : 10×3=30

(a) In a spherical triangle, prove that $\cos a\cos C = \sin a\cot b - \sin C\cot B$. Also, in a spherical triangle if $b + c = \pi$, prove that $\sin 2B + \sin 2C = 0$.

(b) If the colatitude is C, prove that

$$C = x + \cos^{-1}(\cos x \sec y)$$

where $\tan x = \cot\delta \cos H$,

$$\sin y = \cos\delta \sin H$$

H being the hour angle.

(c) If ψ is the angle at the centre of the sun subtended by the line joining two planets at distance a and b from the sun at stationary points, show that

$$\cos\psi = \frac{\sqrt{ab}}{a + b - \sqrt{ab}}$$

(d) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k \tan\xi$, ξ being the apparent zenith distance of a heavenly body.

(e) What is solar eclipse? Mention different types of solar eclipse. Also discuss (with neat diagram) the commencement of solar eclipse.

(f) Discuss the effects of annual parallax on celestial longitude and latitude.

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·4

(Rigid Dynamics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

   (i) What is the moment of inertia of a circular disc of mass M and radius a about a diameter ?

   (ii) State the perpendicular axes theorem on moments of inertia.

(iii) Define radius of gyration of the rigid body about a line.

(iv) Define the centre of percussion.

(v) Define the principal axes of a rigid body at a point O of the body.

(vi) A particle moves on the surface of a sphere. What is the degree of freedom of the particle?

(vii) What do you mean by a conservative mechanical system?

2. Answer the following questions: $2\times4=8$

(a) Prove the perpendicular axes theorem.

(b) A rigid body consists of 3 particles of masses 3 units, 5 units and 2 units located at the points $(-1, 0, 1)$, (2, –1, 3) and (–2, 2, 1) respectively. Find the moments of inertia about

(i) the y-axis, and (ii) the z-axis.

(c) A rigid body of mass 2 units rotates with angular velocity $\vec{\omega} = (1, 1, -1)$ and has the angular momentum $\vec{\Omega} = (2, 3, -1)$. Find the kinetic energy of the body.

(d) A particle of mass 3 units is located at the point (2, 0, 0). The particle rotates about O with angular velocity $\vec{\omega} = \hat{k}$. Find the angular momentum of the particle about O.

3. Answer the following questions : 5×3=15

(a) Find the moment of inertia of a hollow sphere of radius a and mass M about a diameter.

Or

If the moments and products of inertia of a body about three perpendicular concurrent axes are known, find the moment of inertia of the body about the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

(b) State D'Alembert's principle and use it to obtain the equations of motion of any rigid body.

Or

A rough uniform board of mass m and length $2a$ rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Show that the distance through which the board moves in this time is $\frac{2Ma}{M+m}$.

(c) An elliptical lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation, prove that the eccentricity is $\frac{1}{2}$.

Or

Obtain the Lagrangian for a simple pendulum and hence derive the equations of motion of the body.

4. Answer ***any three*** : 10×3=30

(a) Show that the moment of inertia of a right solid cone, whose height is h and the radius of whose base is a, is

$$\frac{3Ma^2}{20}\frac{6h^2+a^2}{h^2+a^2}$$ about a slant side, and

$$\frac{3M}{80}\left(h^2+4a^2\right)$$ about a line through the centre of gravity of the cone perpendicular to its axis.

(b) With usual notation, prove the formula $\frac{1}{2}M\left(V^2+k^2\omega^2\right)$ for the K.E. of a lamina moving in its plane.

Contd.

(c) Define impressed forces and effective forces. A uniform rod OA of length $2a$, free to turn about its end O revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$.

(d) Two equal uniform rods AB and AC, are freely hinged at A and rest in a straight line on a smooth table. A blow is struck at B perpendicular to the rods; show that the kinetic energy generated is $\frac{7}{4}$ times what it would be if the rods were rigidly fastened together at A.

(e) A uniform rod is held in a vertical position with one end resting upon a perfectly rough table, and when released rotates about the end in contact with the table. Find the motion.

(f) Derive the equations of motion of a rigid body in two dimensions when the forces acting on the body are finite.

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3 (Sem–5) MAT M 5

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·5

(Probability)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

   (a) Define probability density function for a continuous random variable.

   (b) What conclusion can be made about the conditional probability $P(A|B)$ if $P(B)=0$?

   (c) State the multiplication theorem of expectation.

Contd.

(d) Write the sample space for the experiment of tossing three coins at a time.

(e) Under what condition $\text{Cov}(X, Y) = 0$?

(f) Choose the correct option for binomial distribution

(i) variance = mean

(ii) variance > mean

(iii) variance < mean

(g) If the mean, median and mode of a continuous distribution coincide, name the distribution.

(h) For a Bernoulli random variable X with $P(X=0)=1-p$ and $P(X=1)=p$, write $E(X)$ and $V(X)$ in terms of p.

2. Answer the following questions : 3×4=12

(a) If X and Y are two random variables, show that

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

(b) Find k, such that the function f defined by

$$f(x)=\begin{cases} kx^2 & , \text{ when } 0<x<1 \\ 0 & , \text{ otherwise} \end{cases}$$

is a probability density function. Also determine $P\left(\frac{1}{3}<x<\frac{1}{2}\right)$.

(c) Show that in a frequency distribution (x_i, f_i); $i=1, 2, \ldots, n$ mathematical expectation of the random variable is nothing but its arithmetic mean.

(d) Define Poisson distribution and hence prove that $\sum_{r=0}^{\infty} p(r)=1$.

3. Answer ***any two*** of the following : 5×2=10

(a) If A and B are independent events, then show that $\overline{A}$ and $\overline{B}$ are also independent events.

(b) A bag contains five balls and it is known how many of these are white. Two balls are drawn and are found to be white. What is the probability that all are white?

(c) A coin is tossed until a head appears. What is the expectation of the number of heads required?

4. Answer ***any two*** of the following : 5×2=10

(a) The probability density function of a two-dimensional random variable (X, Y) is given by

$$\begin{aligned} f(x+y) &= x+y, \ 0 < x+y < 1 \\ &= 0, \text{ elsewhere} \end{aligned}$$

Evaluate $P\left(X < \tfrac{1}{2}, \ Y > \tfrac{1}{4}\right)$

(b) X is a discrete random variable having the following probability mass function :

Mass points x	:	0	1	2	3	4	5	6	7
$P(X = x)$	:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Determine the constant k.

(ii) Find $P(X<6)$.

(iii) What will be $P(X \geq 6)$?

(c) A random variable has the following probability distribution :

x	:	0	1	2	3
$p(x)$	:	0·1	0·3	0·4	0·2

Find (i) $E(X)$, and (ii) $\text{Var}(X)$.

5. Answer **any two** of the following : 5×2=10

(a) If X be a Poisson distributed random variable with the parameter μ, then prove that $E(X)=\mu$ and $V(X)=\mu$

(b) Prove that the mean and variance of a binomially distributed random variable are respectively $\mu = np$ and $\sigma^2 = npq$ (where the symbols have their usual meanings).

(c) Write the probability density function of a random variable X which follows normal distribution with mean μ and variance σ^2. What is standard normal variate ? Find its mean and variance.

6. Answer ***any two*** of the following : 5×2=10

(a) A random variable X has the density function given by

$$f(x) = 2e^{-2x} \quad , \quad x \geq 0$$
$$= 0 \quad , \quad x < 0$$

Find —

(i) the moment generating function;

(ii) the first four moments about the origin.

(b) Find the moment generating function of a continuous probability distribution, whose density is $\frac{1}{2}x^2e^{-x}$, $0 < x < \infty$ and deduce the values of mean and variance.

(c) State and prove Bayes' theorem.

(d) Let the events $A_1, A_2, \ldots, A_k$ be independent and $P(A_i) = p_i$. Show that the probability that at least one of these events will occur is $1 - \prod_{i=1}^{k}(1 - p_i)$.

Total number of printed pages–8

3 (Sem–5) MAT M 6

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·6

(Optimization Theory)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1\times7=7$

(a) The set of all feasible solutions of an LPP is a ———— set.

(Fill in the blank)

Contd.

(b) The set of simultaneous equations

$$x_1 + 2x_2 = 8,\ 3x_1 + x_2 = 9,\ x_1 + x_2 = 4 \text{ is}$$

(i) consistent

(ii) inconsistent

(iii) None of the above

(Choose the correct option)

(c) Given a system of m simultaneous linear equations in n unknowns $(m < n)$, the number of basic variables will be

(i) m

(ii) n

(iii) $n - m$

(iv) $n + m$

(Choose the correct option)

(d) A simplex in n-dimension is a convex polyhedron having

(i) $n - 1$ vertices

(ii) n vertices

(iii) $n + 1$ vertices

(iv) None of the above

(Choose the correct option)

(e) At any iteration of the usual simplex method, if there is at least one basic variable in the basis at zero level and all $z_j - c_j \geq 0$, the current solution is

(i) infeasible

(ii) unbounded

(iii) non-degenerate

(iv) degenerate

(z_j, c_j having usual meaning)

(Choose the correct option)

(f) If the jth primal variable x_j is unrestricted in sign, then the dual problem, the jth constraint is an ________.

(Fill in the blank)

(g) In a balanced transportation problem with m sources and n destinations, the number of constraint equations are ________.

(Fill in the blank)

2. Answer the following questions : 2×4=8

(a) Let S and T be two convex sets in $\mathbb{R}^n$. Then for any scalars $\alpha, \beta \in \mathbb{R}$, $\alpha S + \beta T$ is also convex in $\mathbb{R}^n$.

Contd.

(b) Prove that $x_1=2$, $x_2=-1$ and $x_3=0$ is a solution but not a basic solution to the system of equations

$$3x_1-2x_2+x_3=8$$
$$9x_1-6x_2+4x_3=24$$

(c) Write the dual of the following primal problem :

Minimize $Z=3x_1+5x_2$

subject to $3x_1+5x_2=12$

$4x_1+2x_2=10$

with $x_1, x_2 \geq 0$

(d) Find the extreme points of the convex polygon defined by the inequalities

$$2x_1+x_2+9\geq 0$$
$$-x_1+3x_2+6\geq 0$$
$$x_1+2x_2-3\leq 0$$
$$x_1+x_2\leq 0$$

3. Answer ***any three*** parts of the following : 5×3=15

(a) Solve graphically the following LPP :

Maximize $Z = 5x_1 + 7x_2$

subject to

$$x_1 + x_2 \le 4$$
$$3x_1 + 8x_2 \le 24$$
$$10x_1 + 7x_2 \le 35$$
$$x_1, x_2 \ge 0$$

(b) Prove that the set of all convex combinations of finite number of points of $S \subset \mathbb{R}^n$ is a convex set.

(c) Find all the basic solutions of the equations

$$2x_1 + 3x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 2x_3 = 5$$

and prove that one set of solution is not feasible.

(d) Prove that any convex combination of K different optimum solutions to an LPP

Max $Z = CX,\ C, X^T \in \mathbb{R}^n$

subject to $AX = b$

$$X \ge 0$$

is again an optimum solution to the LPP.

(e) Write the dual of the following primal problem and solve :

$$\text{Min } Z = x_1 + 2x_2$$

subject to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$\text{with } x_1, x_2 \geq 0$$

4. Solve the following LPP by simplex method :

Maximize $Z = 3x_1 + 2x_2 + 5x_3$ 10

subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$

with $x_1, x_2, x_3 \geq 0$

Or

Use the two-phase simplex method to solve

Max $Z = 5x_1 - 4x_2 + 3x_3$

subject to the constraints

$$2x_1 + x_2 - 6x_3 = 20$$
$$6x_1 + 5x_2 + 10x_3 \leq 76$$
$$8x_1 - 3x_2 + 6x_3 \leq 50$$

with $x_1, x_2, x_3 \geq 0$ 10

5. Use duality to solve the following : 10

Minimize $Z = 3x_1 + x_2$

subject to

$$2x_1 + 3x_2 \geq 2$$
$$x_1 + x_2 \geq 1$$
$$\text{with } x_1, x_2 \geq 0$$

Or

If $\hat{x}$ is any feasible solution to the primal

Minimize $Z_p = \overline{c}\overline{x}$

subject to

$$A\overline{x} \geq \overline{b}$$
$$\overline{x} \geq 0$$

and $\hat{w}$ is any feasible solution to the dual

Maximize $Z_D = \overline{b}'\overline{w}$

subject to

$$A'\overline{w} \leq \overline{c}'$$
$$\overline{w} \geq 0$$

then show that, $\overline{c}\overline{x} \geq \overline{b}'\overline{w}$ 10

Contd.

6. Solve the following assignment problem : 10

		Projects			
		A	*B*	*C*	*D*
Engineer	1	12	10	10	8
	2	14	Not suitable	15	11
	3	6	10	16	4
	4	8	10	9	7

Or

Find a solution of the following transportation problem which will minimize the total cost : 10

$D \rightarrow$ $O \downarrow$	D_1	D_2	D_3	D_4	Available a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Requirement b_j	20	40	30	10	