2019

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 2×4=8

(a) For what type of events A and B—

(i) $P(A \cup B) = P(A) + P(B)$;

(ii) $P(A \cap B) = P(A)\,P(B)$? 1+1=2

(b) *(i)* Find c, so that the function

$$f(x) = \begin{cases} cx, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

is a density function.

(ii) Point out the error in the statement "the probability that a student will commit exactly one mistake is 0·5 and the probability that he will commit at least one mistake is 0·03". 1+1=2

(c) Define the following : 1+1=2

(i) Random variable (r.v.)

(ii) Mathematical expectation of an r.v.

(d) Write the condition on—

(i) n, the number of trials;

(ii) p, the probability of success;

so that the Poisson distribution can be obtained as a limiting case of binomial distribution. 1+1=2

2. Answer any *four* of the following questions : 3×4=12

(a) Find the probability of not getting a 7 or 10 total on either of two tosses of a pair of fair dice.

(*Continued*)

(b) Prove or disprove, the second moment about any point a is minimum when taken about the mean μ, i.e.,

$$E(X-a)^2 \geq E(X-\mu)^2$$

(c) A box contains 7 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and it is noted that the 2nd one is white. What is the probability that the first one is also white?

(d) 8 coins are tossed at a time, 256 times. Find the expected frequencies of success (getting a head).

(e) Show that the expectation of a discrete random variable X whose probability function is given by

$$f(x) = \left(\frac{1}{2}\right)^x; \quad x = 1, 2, 3, \cdots$$

is 2.

(f) Prove that the mean of a binomially distributed random variable is $\mu = np$.

(*Turn Over*)

3. Answer any *two* of the following questions : 5×2=10

(a) State and prove Bayes' theorem.

(b) A bag contains 5 white and 3 black balls, another bag contains 4 white and 5 black balls. From any one of these bags, a single draw of 2 balls is made. Find the probability that one of them would be white and the other black ball.

(c) The probabilities of n independent events are $p_1, p_2, \cdots, p_n$. Find the expression for the probability that at least one of the events will happen.

4. Answer any *two* of the following questions : 5×2=10

(a) A random variable X has the density function

$$f(x) = \frac{c}{x^2+1}$$

where $-\infty < x < \infty$.

(i) Find the value of c.

(ii) Find the probability that X^2 lies between $\frac{1}{3}$ and 1.

(b) The joint density function of two continuous random variables X and Y is

$$f(x, y) = \begin{cases} cxy, & 0 < x < 4, 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of c.

(ii) Find $P(X \geq 3, Y \leq 2)$.

(c) Two random variables X and Y are jointly distributed as follows :

$$f(x, y) = \frac{2}{\pi}(1 - x^2 - y^2);\ 0 < x^2 + y^2 < 1$$

Find the marginal distribution of X.

5. Answer any *two* of the following questions : 5×2=10

(a) Prove that the mathematical expectation of the product of two independent random variables is equal to product of their expectations.

(b) Prove that

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}[E(X|Y)]$$

(c) A random variable X has density function given by

$$f(x) = \begin{cases} 2e^{-2x}; & x \geq 0 \\ 0 \ ; & x < 0 \end{cases}$$

Find the—

(i) moment-generating function;

(ii) first four moments about the origin.

6. Answer any *two* of the following questions : 5×2=10

(a) In a binomial distribution, show that the variance is always less than the mean.

(b) What is meant by standard normal variate? Find the mean and variance of standard normal variate.

(Continued)

(c) Find the probability that in a sample of 10 tools chosen at random, exactly two will be defective by using the—

(i) binomial distribution;

(ii) Poisson approximation to the binomial distribution.

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2019

MATHEMATICS

(Major)

Paper : 5.6

(Optimization Theory)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×7=7

(a) If all the constraints are ≥ inequalities in a linear programming problem whose objective function is to be maximized, then the solution of the problem is unbounded.

(State True or False)

(b) If two constraints do not intersect in the positive quadrant of the graph, then

(i) the problem is infeasible

(ii) the solution is unbounded

(iii) one of the constrains is redundant

(iv) None of the above

(Choose the correct option)

(c) Define convex set.

(d) The solution to a transportation problem with m rows and n columns is feasible, if number of positive allocations is

(i) $m+n$

(ii) $m \times n$

(iii) $m+n-1$

(iv) $m+n+1$

(Choose the correct option)

(e) Any two isoprofit or isocost lines for a general LPP are perpendicular to each other.

(State True or False)

(f) A maximization assignment problem is transformed into a minimization problem by

(i) adding each entry in a column with the maximum value in that column

(ii) subtracting each entry in a column from the maximum value in that column

(iii) subtracting each entry in a column from the maximum value in that table

(iv) None of the above

(Choose the correct option)

(g) In a linear programming, all relationships among the decision variables are ______.

(Fill in the blank)

2. Answer the following questions :

(a) Define slack and surplus variables in an LPP. 1+1=2

(b) Define convex hull of a given set $S \subseteq \mathbb{R}^n$. Graph the convex hull of the points (0, 0), (0, 1), (1, 2) and (4, 0). 1+1=2

(c) What are the characteristics of the standard form of an LPP? 2

(d) Prove that the intersection of two convex sets is also a convex set. 2

3. Answer any *three* of the following questions : 5×3=15

(a) An electric company produces two products P_1 and P_2. Products are produced and sold on a weekly basis. The weekly production cannot exceed 25 for product P_1 and 35 for product P_2 because of limited available facilities. The company employs total of 60 workers. Product P_1 requires 2 manweeks of labour, while P_2 requires one man-week of labour. Profit margin on P_1 is ₹ 60 and on P_2 is ₹ 40.

Formulate this problem as an LPP.

(b) Prove that if the i-th constraint in the primal is an equality, then the i-th dual variable is unrestricted in sign.

(c) Prove that a necessary and sufficient condition for the existence of a feasible solution to a transportation problem is that the total capacity (or supply) must be equal to the total requirement (or demand).

(d) Use the graphical method to solve the following LPP :

Maximize $Z = 300x_1 + 400x_2$

subject to the constraints

$$5x_1 + 4x_2 \le 200$$
$$3x_1 + 5x_2 \le 150$$
$$5x_1 + 4x_2 \ge 100$$
$$8x_1 + 4x_2 \ge 80$$

and $x_1, x_2 \ge 0$

(e) Obtain all the basic solutions to the following system of linear equations :

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

4. Solve the following LPP by simplex method : 10

Maximize $Z = 16x_1 + 17x_2 + 10x_3$

subject to the constraints

$$x_1 + x_2 + 4x_3 \le 2000$$
$$2x_1 + x_2 + x_3 \le 3600$$
$$x_1 + 2x_2 + 2x_3 \le 2400$$
$$x_1 \le 30$$

and $x_1, x_2, x_3 \ge 0$

Or

Use Big-M method to solve the following LP problem :

Minimize $Z = 5x_1 + 3x_2$

subject to the constraints

$$2x_1 + 4x_2 \le 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \ge 10$$

and $x_1, x_2 \ge 0$

5. Show that the dual of the dual is the primal. Obtain the dual LP problem of the following primal LP problem : 5+5=10

Minimize $Z = x_1 + 2x_2$

subject to the constraints

$$2x_1 + 4x_2 \le 160$$
$$x_1 - x_2 = 30$$
$$x_1 \ge 10$$

and $x_1, x_2 \ge 0$

Or

State and prove the fundamental duality theorem. 2+8=10

6. A company has three production facilities S_1, S_2 and S_3 with production capacity of 7, 9 and 18 units per week of a product respectively. These units are to be shipped to four warehouses D_1, D_2, D_3 and D_4 with requirement of 5, 8, 7 and 14 units per week respectively. The transportation costs (in ₹) per unit between the factories to warehouses are given in the table below :

	D_1	D_2	D_3	D_4	*Supply* (Availability)
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand (Requirement)	5	8	7	14	34

Formulate this transportation problem as a linear programming model to minimize the total transportation cost. Use North-West corner method to find an initial basic feasible solution to the above transportation problem. 10

Or

A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the following effectiveness matrix :

	I	*II*	*III*	*IV*	*V*
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours?

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