2018

MATHEMATICS

(Major)

Paper : 5.1

(Real and Complex Analysis)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Evaluate :

$$\lim_{(x,\,y)\to(0,\,0)} \frac{x^2y^2}{x^2y^2+(x^2-y^2)^2}$$

(b) Find the infimum of all upper sums of the function $f(x)=3x+1$ on the interval [1, 2].

(c) When is an improper integral said to be convergent?

(d) Define uniform continuity of a function whose domain and codomain are set of complex numbers.

(e) Justify whether true of false :

"If a complex valued function $f(z)$ is analytic, then the real part of $f(z)$ is harmonic."

(f) Verify whether the transformation $w = z^3$ is conformal or not at all points of the region $|z| < 1$.

(g) Write the physical effect of a region transformed from z-plane to w-plane under the transformation $w = az + b$; a, b are given complex constants.

2. Answer the following questions : 2×4=8

(a) Show that the following function is discontinuous at the origin :

$$f(x, y) = \begin{cases} \dfrac{1}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(b) Prove that for a bounded function f

$$\underline{\int_a^b} f\,dx \leq \overline{\int_a^b} f\,dx$$

(Symbols have their usual meaning.)

(c) Test the convergence of

$$\int_0^1 \frac{dx}{\sqrt{1 - x^3}}$$

(d) Prove that the cross ratio is an invariant quantity under bilinear transformation.

3. Answer any *three* parts : 5×3=15

(a) Prove that if f_x and f_y are both differentiable at a point (a, b) of the domain of definition of a function f, then

$$f_{xy}(a, b) = f_{yx}(a, b)$$

(Symbols have their usual meaning.)

(b) Prove that a monotonic function on a closed interval is integrable therein.

(c) Show that the integral

$$\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$$

exists, iff $n < m+1$.

(d) Let $f(z) = u + iv$, z is a complex number, be analytic in a region R. Prove that

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

(e) Let $f(z)$, z is a complex number, be analytic inside and on the boundary C of a simply connected region R. Prove that

$$f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2} dz$$

4. Answer *either* (a) *or* (b) :

(a) (i) Prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}$$

is invariant for change of rectangular axes. 5

(ii) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$$ 5

(b) (i) Show that the function f defined as

$$f(x) = \frac{1}{2^n}$$

when $\frac{1}{2^{n+1}} < x \le \frac{1}{2^n}$, $f(0) = 0$ is integrable on $[0, 1]$. 5

(ii) If f and g are both differentiable on $[a, b]$ and if f' and g' are both integrable on $[a, b]$, then prove that

$$\int_a^b f(x)\, g'(x)\, dx = [f(x)\; g(x)]_a^b - \int_a^b g(x)\, f'(x)\, dx$$ 5

5. Answer *either* (a) *or* (b) :

(a) (i) Prove that if f is bounded and integrable on $[a, b]$, then $|f|$ is also bounded and integrable on $[a, b]$ but the converse is not true. 5

(ii) Find a bilinear transformation that maps points $z = 0, -i, -1$ into $w = i, 1, 0$, respectively. 5

(b) (i) For what value of m and n is the integral

$$\int_0^1 x^{m-1}(1-x)^{n-1}\log x\,dx$$

convergent? 5

(ii) Show that if f and g are positive in $[a, x]$ and

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=l$$

where l is a non-zero finite number, then the two integrals

$$\int_A^\infty f\,dx \text{ and } \int_a^\infty g\,dx$$

converge or diverge together. 5

6. Answer *either (a) or (b)* :

(a) (i) Prove that $f(z)=z^3$ is uniformly continuous but

$$f(z)=\frac{1}{z^3}$$

is not uniformly continuous in the region $|z|<1$. 5

(ii) Find a function v such that $f(z)=u+iv$, z is a complex number, is analytic, where

$$u=x^2-y^2-2xy-2x+3y$$ 5

(b) (i) Evaluate $\int_C \bar{z}\,dz$ along the curve C given by the line from $z=0$ to $z=3i$ and then the line from $z=3i$ to $z=6+3i$. 5

(ii) Evaluate

$$\oint_C \frac{z^2}{(z-1)(z-2)}dz$$

and $\oint_C \frac{e^{2z}}{(z+1)^4}dz$, where C is the circle $|z|=3$. 3+2=5

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MATHEMATICS

(Major)

Paper : 5.2

(Topology)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Describe the open spheres for any discrete metric space (X, d).

(b) Find the derived sets of the following subsets of $\mathbb{R}$:

$$A =]0, 1], \quad B = \left\{\frac{2n+1}{n} : n \in \mathbb{N}\right\}$$

$$C = \left\{-\frac{1}{n} : n \in \mathbb{N}\right\}$$

(c) Define a Cauchy sequence in a metric space (X, d).

(d) Define a topological space and give one example.

(e) Let

$X = \{a, b, c\}$ and

$\mathscr{T} = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$

is a topology on X. Find the derived set of $A = \{a, b\}$.

(f) Let $\mathscr{T}$ be the topology on $\mathbb{N}$ which consists of ϕ and all subsets of the form $G_m = \{m, m+1, m+2, \ldots\}$, $m \in \mathbb{N}$. What are the open sets containing 4?

(g) What do you mean by a Banach space? Give one example.

2. Answer the following questions : 2×4=8

(a) Show that every closed interval is a closed set in the usual metric on $\mathbb{R}$.

(b) Let f be a mapping from $\mathbb{R}$ into $\mathbb{R}$ defined by

$$f(x) = \begin{cases} -2 & \text{when } x < 0 \\ 2 & \text{when } x \geq 0 \end{cases}$$

Examine whether f is continuous with respect to the usual topology on $\mathbb{R}$.

(c) Let $(X, \|\cdot\|)$ be a normed linear space and $x_n \to x$ and $y_n \to y$ in X. Show that $x_n + y_n \to x + y$.

(d) Prove the parallelogram law in an inner product space $(X, \langle \cdot, \cdot \rangle)$.

3. Answer the following questions : 5×3=15

(a) Let (X, d) be a metric space and A and B be subsets of X. Prove that—

(i) $A \subset B \Rightarrow D(A) \subset D(B)$

(ii) $D(A \cup B) = D(A) \cup D(B)$

(b) Let X be any set and $\mathscr{T}$ be the collection of all those subsets of X whose complements are finite together with the empty set. Show that $\mathscr{T}$ is a topology on X. What do you call this topology?

Or

Let $(X, \mathscr{T})$ be a topological space and $A \subset X$. Prove that $\bar{A} = A \cup D(A)$.

(c) Show that $\mathbb{R}^n$ is a normed linear space with some suitable norm.

Or

Let $(X, \langle \cdot, \cdot \rangle)$ be an inner product space. Prove that for all $x, y \in X$

$$4\langle x, y \rangle = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$$

4. Answer the following questions : 10×3=30

(a) Prove that every non-empty open set on the real line is the union of a countable class of pairwise disjoint open intervals.

Or

State and prove Cantor's intersection theorem for metric spaces.

(b) Let (X, d) be a metric space and $x_0 \in X$ be fixed. Show that the real-valued function $f_{x_0}(x) = d(x, x_0)$, $x \in X$ is continuous. Is it uniformly continuous? Let (Y, P) be another metric space and $f : X \to Y$ be a mapping. Prove that f is continuous if and only if the inverse image of every open set in Y is an open set in X. 2+1+7

Or

Let X be a metric space and Y be a complete metric space. Let A be a dense subspace of X. If $f : A \to Y$ is uniformly continuous, then prove that f can be extended uniquely to a uniformly continuous mapping $g : X \to Y$.

(c) Prove that a metric space is compact if and only if it is complete and totally bounded.

Or

Let $\{A_\lambda : \lambda \in \Lambda\}$ be a family of connected subsets of a space X such that

$$\bigcap_{\lambda \in \Lambda} A_\lambda \neq \phi$$

Prove that $\bigcup_{\lambda \in \Lambda} A_\lambda$ is a connected set in X.

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2018

MATHEMATICS

(Major)

Paper : 5.3

(Spherical Trigonometry and Astronomy)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

   (a) State one fundamental difference between a spherical triangle and a plane triangle.

   (b) Define polar triangle and its primitive triangle.

   (c) Mention one property of pole of a great circle.

   (d) What is the reason of the oval shape of the sun at rising?

   (e) Explain briefly the dynamical significance of Kepler's second law of motion.

   (f) Define orbital period and synodic period of a planet.

   (g) What is the declination of the pole of the ecliptic?

(Turn Over)

2. Answer the following questions : 2×4=8

(a) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

(b) Prove that section of a sphere by a plane is a circle.

(c) Show that right ascension α and declination δ of the sun is always connected by the equation

$$\tan\delta = \tan\varepsilon \sin\alpha$$

ε being obliquity of the ecliptic.

(d) The apparent altitude of a star due to refraction is 30°. Calculate the true altitude, the coefficient of refraction being 58·2″.

3. Answer any *three* questions of the following : 5×3=15

(a) A port is in latitude l (north) and longitude λ (west). Show that the longitudes of places on the equator distance δ from the port are

$$\lambda \pm \cos^{-1}(\cos\delta \sec l)$$

(b) What do you mean by rising and setting of a star? Prove that the hour angle H of a star at the time of setting is given by

$$\cos H = -\tan\phi \tan\delta$$

(Continued)

(c) Prove that

$$\cos\nu = \frac{\cos E - e}{1 - e\cos E} \text{ and } \sin\nu = \frac{\sqrt{1-e^2}\sin E}{1-e\cos E}$$

where ν is the true anomaly and E is the eccentric anomaly at any position of a planet in its orbit.

(d) If λ is the moon's celestial latitude at the instant of opposition, m and p her hourly motions in longitude and latitude respectively, s the hourly motion of the sun in longitude and C the sum of semi-diameters of the moon and that of the earth's shadow, show that the duration of the lunar eclipse is the difference between the two roots of t, given by

$$C^2 = (\lambda - pt)^2 + (m-s)^2 t^2$$

(e) Define geocentric parallax. Show that geocentric parallax of a heavenly body varies as the sine of its apparent zenith distance.

4. Derive cosine formula related to a spherical triangle. In an equilateral spherical triangle ABC, prove the following :

(i) $2\cos\frac{a}{2}\cdot\sin\frac{A}{2} = 1$

(ii) $\sec A = 1 + \sec a$ 6+4=10

(Turn Over)

5. *(a)* Derive the formula for refraction

$$R = k \tan \zeta$$

ζ being the apparent zenith distance of a heavenly body. Mention one limitation of this formula. 5+1=6

(b) If z_1 and z_2 are the zenith distances of a star at upper and lower culmination respectively which are on opposite sides of the zenith, prove that

$$\delta = 90° - \frac{z_1 + z_2}{2} \text{ and } \phi = 90° - \frac{z_2 - z_1}{2}$$

where δ is the declination of the star and ϕ is the latitude of the place of observer. 4

6. Define solar ecliptic limits. Show that the minimum angular distance D_0 of the moon and the sun for occurrence of solar eclipse will be

$$D_0 = \beta \cos j$$

where $\tan j = \frac{\tan i}{1-m}$ the other symbols carry usual meanings. 2+8=10

Or

Discuss the effects of annual parallax on celestial longitude and latitude. 10

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2018

MATHEMATICS

(Major)

Paper : 5.4

(Rigid Dynamics)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×7=7

(a) Write down the moment of inertia of a circular disc of mass M and radius a about an axis through its centre and perpendicular to its plane.

(b) Define radius of gyration of the rigid body about a line.

(c) State the principal axes of a rigid body at a point O of the body.

(d) A rigid body rotates with angular velocity $\vec{\omega}$ about a fixed axis and I denotes the moment of inertia of the body about the axis. Write down the expression for the kinetic energy of the body.

(e) What do you mean by holonomic system?

(f) Define conservative system.

(g) State the theorem of the principle of conservation of energy of a rigid body.

2. Answer the following questions : $2 \times 4 = 8$

(a) A rigid body consists of 3 particles of masses 3 units, 5 units and 2 units located at the points $(-1, 0, 1)$, $(2, -1, 3)$ and $(-2, 2, 1)$ respectively. Find the moments of inertia about (i) the y-axis and (ii) the z-axis.

(b) A body with one point fixed rotates with angular velocity $(0, 0, 2)$. Find the magnitude of the velocity of a particle of mass m of the body located at the point $(3, -4, 1)$.

(c) Find the number of degrees of freedom for a rigid body which has one point fixed but can move in space about this point.

(d) A rigid body of mass 2 units rotates with angular velocity $\vec{\omega} = (1, 1, -1)$ and has the angular momentum $\vec{\Omega} = (2, 3, -1)$. Find the kinetic energy of the body.

3. Answer the following questions : $5 \times 3 = 15$

(a) Find the moment of inertia of a hollow sphere of radius a and mass M about a diameter.

Or

If the moments and products of inertia of a body about three perpendicular concurrent axes are known, find the moment of inertia of the body about the line

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

(b) State d'Alembert's principle and use it to obtain the equations of motion of any rigid body.

Or

Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

(c) The lengths AB and AD of the sides of a rectangle $ABCD$ are $2a$ and $2b$ respectively. Show that the inclination to AB of one of the principal axes at A is

$$\frac{1}{2}\tan^{-1}\frac{3ab}{2(a^2 - b^2)}$$

4. Define impressed forces and effective forces. A uniform rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a smooth joint at one extremity of the rod so that it describes a cone of semivertical angle α. Show that

$$\omega^2 = \frac{3g}{4a\cos\alpha}$$ 2+8=10

Or

(a) A plank of mass m and length $2a$ is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M, starting from the upper end walks down the plank so that it does not move, show that he will reach the other end in time

$$\sqrt{\frac{4Ma}{(m+M)g\sin\alpha}}$$ 6

(b) A rod revolving on a smooth horizontal plane about one end, which is fixed, breaks into two parts; what is the subsequent motion of the two parts?

5. (a) A pendulum is supported at O and P is the centre of oscillation. Show that if an additional weight is rigidly attached at P, the period of oscillation is unaltered.

(b) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

6. Derive the equations of motion of a rigid body in two dimensions when the forces acting on the body are finite.

(Continued

Or

Write down the equations of motion of a rigid body in two dimensions under impulsive forces. Two equal uniform rods, AB and AC, are freely jointed at A, and are placed on a smooth table so as to be at right angles. The rod AC is struck by a blow at C in a direction perpendicular to itself. Show that the resulting velocities of the middle points of AB and AC are in the ratio 2 : 7. 3+7=10

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2018

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×8=8

(a) Write the condition for the outcomes of a random experiment so that $p+q=1$; p and q being the probability of success and failure respectively.

(b) If S is the sample space in a random toss of 7 coins, then write the number of elements of S.

(c) Is the probability mass function

x	-1	0	1
$p(x)$	$0 \cdot 4$	$0 \cdot 4$	$0 \cdot 3$

admissible? Give reason.

(d) Sketch the area under any probability curve with probability density function $p(x)$ between $x = c$ and $x = d$ represented by $P(c \leq X \leq d) = \int_c^d p(x)\, dx$.

(e) For a discrete random variable X with probability function $p(x)$, rth moment about A is $\sum (x - A)^r p(x)$. What are the values of r and A for (i) $E(X)$ and (ii) var (X)?

(f) The density function of a random variable X is given by

$$f(x) = \begin{cases} 2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find $E(X)$.

(g) What is the probability of getting exactly 3 heads in 6 tosses of a fair coin?

(h) Name the distribution in which mean is the square of its standard deviation.

2. Answer any *four* of the following : 3×4=12

(a) If A and B are two possible outcomes of an experiment and $p(A) = 0 \cdot 4$, $P(A \cup B) = 0 \cdot 7$ and $P(B) = p$, then for what value of p, A and B become independent?

(b) A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Evaluate $P(X \geq 6)$ and $P(0 < x < 5)$.

(c) Show that in a frequency distribution

$$(x_i, f_i);\ i = 1, 2, \cdots, n$$

mathematical expectation of the random variable is nothing but its arithmetic mean.

(d) Define Poisson distribution and hence prove that $\sum_{r=0}^{\infty} p(r) = 1$.

(e) If the random variable X is normally distributed with mean μ and variance σ^2, show that the mean of the variate

$$z = \frac{x - \mu}{\sigma}$$

is always zero.

3. Answer any *two* from the following : 5×2=10

(a) Prove that two events A and B are independent $\Leftrightarrow P(A \cap B) = P(A)\, P(B)$.

(b) A man has five coins, one of which has two heads. He randomly takes out a coin and tosses in three times. What is the probability that it will fall head upward all the times?

(c) For two independent events A and B, prove that *(i)* A and $\overline{B}$ are independent and *(ii)* $\overline{A}$ and $\overline{B}$ are independent.

4. Answer any *two* from the following : 5×2=10

(a) Let X and Y be two random variables each taking three values −1, 0, 1 and having the joint probability distribution as given in the following table :

Y \ X	−1	0	1
−1	0	·1	·1
0	·2	·2	·2
1	0	·1	·1

Obtain the marginal probability distribution of X and Y.

(b) The probability function of a random variable X is given by

$$f(x) = \begin{cases} x^2/81, & -3 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability density function for the random variable

$$u = \frac{1}{3}(12 - X)$$

(c) A random variable X has density function

$$f(x) = \begin{cases} ce^{-3x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

Find (i) the constant c, (ii) $P(1 < x < 2)$ and (iii) $P(X \ge 3)$.

5. Answer any *two* from the following : 5×2=1(

(a) Prove that

$$\text{var}(ax + by) = a^2 \text{ var}(x) + b^2 \text{ var}(y) + 2ab \text{ cov}(x, y)$$

(b) A random variable has the following probability distribution :

x	0	1	2	3
$p(x)$	0·1	0·3	0·4	0·2

Find (i) $E(X)$ and (ii) $\text{var}(X)$.

(c) A continuous random variable X has the probability function given by

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

Find (i) $E(X)$ and (ii) $E(X^2)$.

6. Answer any *two* from the following : 5×2=10

(a) Prove that for the binomial distribution with parameter n and p, variance cannot exceed $\frac{n}{4}$.

(b) Derive Poisson distribution as a limiting case of binomial distribution.

(c) Prove that the mean and variance of a binomially distributed variable are respectively $\mu = np$ and $\sigma^2 = npq$.

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2018

MATHEMATICS

(Major)

Paper : 5.6

(Optimization Theory)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed : 1×7=7

(a) Given a system of m simultaneous linear equations in n unknowns $(m < n)$, the number of basic variables will be

(i) m

(ii) n

(iii) $n - m$

(iv) $n + m$

(Choose the correct option)

(b) Express the vector $x = (5, 9)$ as the linear combination of the vectors $\alpha = (1, 2)$, $\beta = (3, 4)$.

(c) Define a line segment joining the points x and y in $\mathbb{R}^2$.

(d) The set of all feasible solutions of an LPP is a _____ set.

(Fill in the blank)

(e) In standard form of an LPP, all the constraints are expressed in the form of equations, except for the non-negative restrictions.

(State True or False)

(f) A necessary and sufficient condition for BFS to a maximization LPP to be an optimum is (for all j)

(i) $z_j - c_j \geq 0$

(ii) $z_j - c_j \leq 0$

(iii) $z_j - c_j = 0$

(iv) $z_j - c_j > 0$ or < 0

(Choose the correct option)

(Continued)

(g) Which of the following is not a convex set?

(i) $\{(x_1, x_2) : x_1^2 + x_2^2 = 1\}$

(ii) $\{(x_1, x_2) : |x_1| \leq 1, |x_2| \leq 1\}$

(iii) $\{(x_1, x_2) : x_1^2 + (x_2 - 1)^2 \leq 4\}$

(iv) None of the above

(Choose the correct option)

2. Answer the following questions : 2×4=8

(a) Show that a hyperplane in $\mathbb{R}^n$ is a convex set.

(b) Define the convex hull of a set $A \subseteq \mathbb{R}^n$. Determine the convex hull of the set $A = \{x_1, x_2\}$.

(c) Prove that $x_1 = 2$, $x_2 = -1$ and $x_3 = 0$ is a solution but not a basic solution to the system of equations

$$3x_1 - 2x_2 + x_3 = 8$$
$$9x_1 - 6x_2 + 4x_3 = 24$$

(Turn Over)

(d) Write the dual of the following primal problem :

$$\text{Minimize } Z = 5x_1 + 3x_2$$

subject to

$$3x_1 + 5x_2 = 12$$
$$5x_1 + 2x_2 = 10$$

with $x_1 \geq 0,\ x_2 \geq 0$

3. Answer any *three* parts of the following : 5×3=15

(a) Three different types of trucks A, B and C have been used to transport a minimum of 60 tons solid and 35 tons liquid substance. A type truck can carry 7 tons solid and 3 tons liquid. B type truck can carry 6 tons solid and 2 tons liquid and C type truck can carry 3 tons solid and 4 tons liquid. The costs of transport are ₹500, ₹400 and ₹450 per truck of A, B and C type respectively. Formulate the problem mathematically so that the total transportation cost is minimum.

(b) What is a balanced transportation problem? Describe a transportation table. Write the names of three common methods to obtain an initial basic feasible solution for a transportation problem. 1+1+3=5

(Continued)

(c) Solve graphically the following linear programming problem :

$$\text{Maximize } Z = 5x_1 + 7x_2$$

subject to

$$3x_1 + 8x_2 \leq 12$$
$$x_1 + x_2 \leq 2$$
$$2x_1 \leq 3$$

with $x_1 \geq 0,\ x_2 \geq 0$

(d) Prove that the set of all convex combinations of a finite number of points of $S \subseteq \mathbb{R}^n$ is a convex set.

(e) Find out all the basic solutions of the equations :

$$2x_1 + 3x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 2x_3 = 5$$

and prove that one set of solution is not feasible.

4. Solve the following LPP by simplex method : 10

$$\text{Maximize } Z = 3x_1 + 2x_2 + 5x_3$$

subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$

with $x_1, x_2, x_3 \geq 0$

(Turn Over)

Or

Solve the following by two-phase method :

Maximize $Z = 5x_1 + 3x_2$

subject to

$$3x_1 + x_2 \le 1$$
$$3x_1 + 4x_2 \ge 12$$

with $x_1, x_2 \ge 0$

5. Use Charnes Big-M method to solve the following LPP :

Maximize $Z = 3x_1 - x_2$

subject to

$$2x_1 + x_2 \ge 2$$
$$x_1 + 3x_2 \le 3$$
$$x_2 \le 4$$

with $x_1, x_2 \ge 0$

Or

Use duality to solve the following :

Minimize $Z = 3x_1 + x_2$

subject to

$$2x_1 + 3x_2 \ge 2$$
$$x_1 + x_2 \ge 1$$

with $x_1, x_2 \ge 0$

(*Continued*)

6. Solve the following transportation problem by using Vogel's approximations method for determination of IBFS and show that the optimal solution is degenerate : 10

	D_1	D_2	D_3	D_4	a_i
O_1	10	20	5	7	15
O_2	18	9	12	8	25
O_3	15	14	16	18	5
b_j	5	15	15	10	

Or

A company has 4 machines to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table :

		Machine			
		W	*X*	*Y*	*Z*
Job	*A*	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

Assign the jobs to different machines so as to minimize the total cost. 10

★★★