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3 (Sem–1/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics – I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : $1\times7=7$

   (a) Define unit vectors.

   (b) If $\vec{A}.\vec{B}=0$, then what is the angle between $\vec{A}$ and $\vec{B}$?

   (c) What is a 'DEL' operator ?

   (d) Find the Laplacian of the scalar field $\phi = xy^2z^3$

Contd.

(e) State Green's theorem.

(f) Write the order and degree of the differential equation

$$2y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^4=0$$

(g) What do you understand by the statement $\vec{\nabla}.\vec{A}=0$?

(h) What is an 'error' in statistics ?

(i) Define coordinate surfaces in curvilinear co-ordinates.

(j) Write the integrating factor of the differential equation

$$\frac{dy}{dx}+5y=x^2$$

(k) Write the geometrical interpretation of the scalar triple product.

(l) Define variance in statistics.

2. Answer ***any four*** of the following questions : 2×4=8

(a) Give examples of a scalar field and a vector field.

(b) If $\vec{r}$ represents the position vector, then find the value of $\vec{\nabla}.\vec{r}$.

(c) Define the line integral of a vector.

(d) Write down the relation of cylindrical co-ordinate (r,θ,z) with cartesian co-ordinate (x,y,z).

(e) Explain the scale factors h_1, h_2, h_3 in curvilinear co-ordinate system.

(f) For what value of N, the vectors $\vec{A}=2\hat{i}+3\hat{j}-6\hat{k}$ and $\vec{B}=N\hat{i}+2\hat{j}+2\hat{k}$ are perpendicular to each other.

(g) Evaluate $\iint_S \vec{r}.\hat{n}\,ds$, where S is a closed surface.

(h) Prove that $\delta(x)=\delta(-x)$.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Show that

$$\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}$$

(b) If $\phi = xy + yz + zx$ and $\vec{F} = \vec{\nabla}\phi$, then find $\vec{\nabla}.\vec{F}$ and $\vec{\nabla} \times \vec{F}$.

(c) Apply Green's theorem in the plane to evaluate the integral

$$\oint_C \left[\left(xy - x^2\right)dx + x^2 y\, dy \right]$$

over the triangle bounded by the lines $y = 0$, $x = 1$ and $y = x$.

(d) Solve the differential equation

$$2xy\frac{dy}{dx} = x^2 + 3y^2$$

(e) Express $\nabla^2\psi$ in cylindrical coordinate system.

(f) Prove that

$$\delta\left(x^2 - a^2\right) = \frac{1}{2a}\left[\delta\left(x - a\right) + \delta\left(x + a\right)\right]$$

(g) A function is defined as

$$f(x) = \begin{cases} 0 \text{ for } x < 2 \\ \frac{1}{18}(2x + 3) \text{for } 2 \le x \le 4 \\ 0 \text{ for } x > 2 \end{cases}$$

Show that it is a probability density function.

(h) If $\vec{F}$ is a vector, prove that

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{F}\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{F}\right) - \nabla^2\vec{F}$$

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Show that the gradient of a scalar field is a vector. 5

(ii) Show that 2½×2=5

1. *div curl* $\vec{A} = 0$ and

2. $curl(grad\,\phi) = 0$

(b) (i) Define curvilinear co-ordinate system. When it is called orthogonal ? 3+1=4

(ii) Obtain expression for length, area and volume elements in curvilinear coordinate system. 2+2+2=6

(c) (i) State and explain Gauss-divergence theorem. 3

(ii) Give the physical meaning of divergence and curl of a vector. 2+2=4

(iii) Find an expression of $\vec{\nabla}.\vec{A}$ in spherical polar co-ordinate system. 3

(d) (i) Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$. 5

(ii) Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$. 5

(e) Solve the following differential equations : 5+5=10

(i) $\left(1 + x^2\right)\frac{dy}{dx} + 2xy = \cos x$

(ii) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$

(f) State and prove Stoke's theorem. Using Stoke's theorem show that

$\oint_C \vec{r} \times d\vec{r} = 2\iint_S d\vec{S}$, where C is the closed perimeter curve bounding the open surface S. 1+5+4=10

(g) (i) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, subject to the condition $y(0) = 0,\ y'(0) = 1$ 6

(ii) Prove that $\vec{A}.\left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \times \vec{B}\right).\vec{C}$ 4

(h) (i) If $\vec{A} = 6\hat{i} + 4\hat{j} + 3\hat{k}$
$\vec{B} = 2\hat{i} - 3\hat{j} - 3\hat{k}$
$\vec{C} = \hat{i} + \hat{j} + \hat{k}$ then evaluate

$\vec{A} \times \left(\vec{B} \times \vec{C}\right)$ 4

(ii) Evaluate $\oint_C x^2y\,dx + y^2dy$, where C is the boundary of the region enclosed by $y = x$ and $y^2 = x$. 6

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3 (Sem – 1 / CBCS) PHY HC 2

2022

PHYSICS

(Honours)

Paper : PHY–HC–1026

(Mechanics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : $1 \times 7 = 7$

   (a) Write *one* limitation of Newton's law of motion.

   (b) What is the relation between workdone and kinetic energy ?

   (c) Define the co-efficient of restitution.

   (d) What do you mean by radius of gyration ?

Contd.

(e) Write the limiting value of Poisson's ratio.

(f) Which of the following is used to calculate the rate of flow of a liquid through a capillary tube?

(i) Stokes' law

(ii) Bernoulli's theorem

(iii) Pascal's law

(iv) Poiseuille's law

(g) State the law of gravitation.

(h) Define Sharpness of resonance.

(i) What is fictitious forces?

(j) Give *one* example of a massless particle.

(k) What is wave number?

(l) Write the relation between torque and angular momentum.

2. Answer ***any four*** of the following questions : 2×4=8

(a) What do you mean by non-conservative force? Give an example with justification.

(b) A $10kg$ ball and $20kg$ ball approaches each other with velocities $20m/sec$ and $10m/sec$ respectively. What are their velocities after collision if the collision is perfectly elastic?

(c) Establish the defining equation of simple harmonic motion.

(d) The co-ordinates of an event in the moving frame S' moving with velocity $12m/sec$ along the x-axis are $(5, 7, 5)$. Find the co-ordinates of the same event in the frame S if their origins co-incides $1/4$ seconds later.

(e) Write the difference between inertial mass and gravitational mass.

(f) What is resonance? Write the condition of resonance.

(g) State Kepler's third law of planetary motion.

(h) Explain how the mass of a body varies with velocity.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Derive the expression of the final velocity of a Rocket considering the value of g is constant.

(b) Draw and explain potential energy curve. What are stable and unstable equilibrium ? 1+3+1=5

(c) Obtain the velocity after one dimensional inelastic collision between two particles in centre of mass frame.

(d) If a uniform rod of material having Poisson's ratio 0·5 suffers a longitudinal strain of 1×10^{-4}, find the % change in its volume.

(e) Discuss how two body problem in central force motion is reduced to one body problem.

(f) Consider a fluid having coefficient of viscosity η and density ρ flowing through a cylindrical tube of radius r and length l. If P is the pressure difference in the liquid at the two ends, show that the volume of fluid flowing in time t is

$$V = \frac{\pi P r^4}{8 \eta l} \cdot t$$

(g) Establish that centrifugal force produced as a result of earth's rotation, is

$$\vec{F} = -m\vec{w} \times (\vec{w} \times \vec{r})$$

where the symbols have their usual meanings.

(h) Write the Lorentz transformation equations. Under what condition the Lorentz transformation equations become Galilean transformation. 3+2=5

4. Answer ***any three*** of the following questions : 10×3=30

(a) Define the different types of frame of reference. Derive the Galilean transformation equation in inertial frame of reference. Show that velocity is variant and acceleration is invariant under Galilean transformation. 2+4+4=10

(b) Point out the difference between conservative and non-conservative forces. Prove that a conservative force $\vec{F}$ is derivable from a potential ϕ, $\vec{F} = -\vec{\nabla}\phi$ and hence obtain $\vec{\nabla} \times \vec{F}$. 2+6+2=10

(c) Define Moment of inertia. Explain the *two* theorem of moment of inertia. Calculate the moment of inertia of a solid sphere about a diameter. 1+2+2+5=10

(d) Derive an expression of acceleration in uniformly rotating frame of reference. Write *any two* applications of Coriolis force. 8+2=10

(e) Define Young's modulus, bulk modulus and rigidity modulus of elasticity.

Deduce the relation

$$\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta},$$ where the symbols have their usual meaning. 3+7=10

(f) What do you mean by gravitational potential and gravitational field intensity. Write their relation. Find out an expression for gravitational potential due to a solid sphere at an inside point. 2+1+7=10

(g) State the basic postulates of special theory of relativity. Deduce Einstein's mass-energy relation $E = mc^2$ and discuss it. 2+6+2=10

(h) Write short notes on ***any two*** of the following : 5×2=10

(i) Length contraction

(ii) Compound pendulum

(iii) Relativistic Doppler effect

(iv) Cantilever

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