3 (Sem-1/CBCS) PHY HC 1

2022 PHYSICS

(Honours)

Paper: PHY-HC-1016

(Mathematical Physics-I)

Full Marks: 60

Time: Three hours

The figures in the margin indicate full marks for the questions.

- 1. Answer **any seven** of the following questions: 1×7=7
 - (a) Define unit vectors.
 - (b) If $\vec{A} \cdot \vec{B} = 0$, then what is the angle between \vec{A} and \vec{B} ?
 - (c) What is a 'DEL' operator?
 - (d) Find the Laplacian of the scalar field $\phi = xy^2z^3$

- (e) State Green's theorem.
- (f) Write the order and degree of the differential equation

$$2y\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^4 = 0$$

- (g) What do you understand by the statement $\nabla \cdot \vec{A} = 0$?
- (h) What is an 'error' in statistics?
- (i) Define coordinate surfaces in curvilinear co-ordinates.
- (j) Write the integrating factor of the differential equation

$$\frac{dy}{dx} + 5y = x^2$$

- (k) Write the geometrical interpretation of the scalar triple product.
- (1) Define variance in statistics.
- 2. Answer **any four** of the following questions: $2\times4=8$
 - (a) Give examples of a scalar field and a vector field.

- (b) If \vec{r} represents the position vector, then find the value of $\vec{\nabla} \cdot \vec{r}$.
- (c) Define the line integral of a vector.
- (d) Write down the relation of cylindrical co-ordinate (r,θ,z) with cartesian co-ordinate (x,y,z).
- (e) Explain the scale factors h_1, h_2, h_3 in curvilinear co-ordinate system.
- (f) For what value of N, the vectors $\vec{A} = 2\hat{i} + 3\hat{j} 6\hat{k}$ and $\vec{B} = N\hat{i} + 2\hat{j} + 2\hat{k}$ are perpendicular to each other.
- (g) Evaluate $\iint_{S} \vec{r} \cdot \hat{n} ds$, where S is a closed surface.
- (h) Prove that $\delta(x) = \delta(-x)$.
- 3. Answer **any three** of the following questions: $5\times3=15$
 - (a) Show that

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

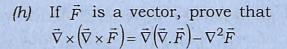
- (b) If $\phi = xy + yz + zx$ and $\vec{F} = \vec{\nabla} \phi$, then find $\vec{\nabla} \cdot \vec{F}$ and $\vec{\nabla} \times \vec{F}$.
- (c) Apply Green's theorem in the plane to evaluate the integral $\oint_C \left[(xy x^2) dx + x^2 y dy \right]$

over the triangle bounded by the lines y = 0, x = 1 and y = x.

- (d) Solve the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$
- (e) Express $\nabla^2 \psi$ in cylindrical coordinate system.
- (f) Prove that $\delta(x^2 a^2) = \frac{1}{2a} [\delta(x a) + \delta(x + a)]$
- (g) A function is defined as

$$f(x) = \begin{cases} 0 & \text{for } x < 2 \\ \frac{1}{18} (2x+3) & \text{for } 2 \le x \le 4 \\ 0 & \text{for } x > 2 \end{cases}$$

Show that it is a probability density function.



- 4. Answer **any three** of the following questions: 10×3=30
 - (a) (i) Show that the gradient of a scalar field is a vector.
 - (ii) Show that $2\frac{1}{2} \times 2 = 5$
 - 1. div curl $\vec{A} = 0$ and
 - 2. $\operatorname{curl}(\operatorname{grad}\phi) = 0$
 - (b) (i) Define curvilinear co-ordinate system. When it is called orthogonal? 3+1=4
 - (ii) Obtain expression for length, area and volume elements in curvilinear coordinate system. 2+2+2=6
 - (c) (i) State and explain Gauss-divergence theorem. 3
 - (ii) Give the physical meaning of divergence and curl of a vector.

 2+2=4
 - (iii) Find an expression of $\nabla \cdot \vec{A}$ in spherical polar co-ordinate system.

(d) (i) Find the directional derivative of
$$\phi(x,y,z) = xy^2 + yz^3$$
 at the point $(2,-1,1)$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

(ii) Prove that
$$\nabla^2 \left(\frac{1}{r}\right) = 0$$
.

(e) Solve the following differential equations: 5+5=10

(i)
$$(1+x^2)\frac{dy}{dx} + 2xy = \cos x$$

(ii)
$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

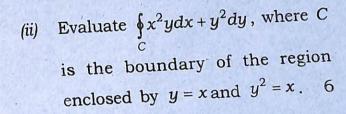
State and prove Stoke's theorem. Using Stoke's theorem show that $\oint_C \vec{r} \times d\vec{r} = 2 \iint_S d\vec{S}, \text{ where } C \text{ is the closed}$ perimeter curve bounding the open surface S.

(g) (i) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, subject to the condition y(0) = 0, y'(0) = 1

ii) Prove that
$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}) \cdot \vec{C}$$

(h) (i) If
$$\vec{A} = 6\hat{i} + 4\hat{j} + 3\hat{k}$$

 $\vec{B} = 2\hat{i} - 3\hat{j} - 3\hat{k}$
 $\vec{C} = \hat{i} + \hat{j} + \hat{k}$ then evaluate
 $\vec{A} \times (\vec{B} \times \vec{C})$





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3 (Sem-1/CBCS) PHY HC 2

2022

PHYSICS

(Honours)

Paper: PHY-HC-1026

(Mechanics)

Full Marks: 60

Time: Three hours

The figures in the margin indicate full marks for the questions.

- 1. Answer **any seven** of the following questions: $1 \times 7 = 7$
 - (a) Write one limitation of Newton's law of motion.
 - (b) What is the relation between workdone and kinetic energy?
 - (c) Define the co-efficient of restitution.
 - (d) What do you mean by radius of gyration?

Contd.

- (e) Write the limiting value of Poisson's ratio.
- (f) Which of the following is used to calculate the rate of flow of a liquid through a capillary tube?
 - (i) Stokes' law
 - (ii) Bernoulli's theorem
 - (iii) Pascal's law
 - (iv) Poiseuille's law
- (g) State the law of gravitation.
- (h) Define Sharpness of resonance.
- (i) What is fictitious forces?
- (j) Give one example of a massless particle.
- (k) What is wave number?
- (1) Write the relation between torque and angular momentum.

- 2. Answer *any four* of the following questions: 2×4=8
 - (a) What do you mean by non-conservative force? Give an example with justification.
 - (b) A 10kg ball and 20kg ball approaches each other with velocities 20m/sec and 10m/sec respectively. What are their velocities after collision if the collision is perfectly elastic?
 - (c) Establish the defining equation of simple harmonic motion.
 - (d) The co-ordinates of an event in the moving frame S' moving with velocity 12m/sec along the x-axis are (5, 7, 5). Find the co-ordinates of the same event in the frame S if their origins co-incides 1/4 seconds later.
 - (e) Write the difference between inertial mass and gravitational mass.
 - (f) What is resonance? Write the condition of resonance.
 - (g) State Kepler's third law of planetary motion.
 - (h) Explain how the mass of a body varies with velocity.

- 3. Answer any three of the following $5 \times 3 = 15$ questions:
 - Derive the expression of the final velocity of a Rocket considering the value of q is constant.
 - Draw and explain potential energy curve. What are stable and unstable 1+3+1=5 equilibrium?
 - Obtain the velocity after one (c) dimensional inelastic collision between two particles in centre of mass frame.
 - If a uniform rod of material having Poisson's ratio 0.5 suffers a longitudinal strain of 1×10^{-4} , find the % change in its volume.
 - Discuss how two body problem in central force motion is reduced to one body problem.
 - Consider a fluid having coefficient of viscosity η and density ρ flowing through a cylindrical tube of radius r and length l. If P is the pressure difference in the liquid at the two ends, show that the volume of fluid flowing in time t is

$$V = \frac{\pi P r^4}{8\eta l} \cdot t$$

Establish that centrifugal force produced as a result of earth's rotation,

$$\vec{F} = -m\vec{w} \times (\vec{w} \times \vec{r})$$
re the symbols have their usu

where the symbols have their usual meanings.

Write the Lorentz transformation equations. Under what condition the Lorentz transformation equations become Galilean transformation.

3+2=5

- Answer any three of the following 10×3=30 questions:
 - (a) Define the different types of frame of reference. Derive the Galilean transformation equation in inertial frame of reference. Show that velocity is variant and acceleration is invariant under Galilean transformation.

2+4+4=10

Point out the difference between conservative and non-conservative forces. Prove that a conservative force \vec{F} is derivable from a potential ϕ , $\vec{F} = -\vec{\nabla}\phi$ and hence obtain $\vec{\nabla} \times \vec{F}$.

2+6+2=10

(c) Define Moment of inertia. Explain the two theorem of moment of inertia. Calculate the moment of inertia of a solid sphere about a diameter.

1+2+2+5=10

- (d) Derive an expression of acceleration in uniformly rotating frame of reference.

 Write any two applications of Coriolis force.

 8+2=10
- (e) Define Young's modulus, bulk modulus and rigidity modulus of elasticity.

 Deduce the relation

 $\frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$, where the symbols have their usual meaning. 3+7=10

- (f) What do you mean by gravitational potential and gravitational field intensity. Write their relation. Find out an expression for gravitational potential due to a solid sphere at an inside point.

 2+1+7=10
- (g) State the basic postulates of special theory of relativity. Deduce Einstein's mass-energy relation $E = mc^2$ and discuss it. 2+6+2=10

- (h) Write short notes on any two of the following: $5\times2=10$
 - (i) Length contraction
 - (ii) Compound pendulum
 - (iii) Relativistic Doppler effect
 - (iv) Cantilever

