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3 (Sem–1 /CBCS) PHY HC 1

2021

(Held in 2022)

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics –I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

   (a) State the vector field with respect to Cartesian co-ordinate. Give *one* example.

   (b) Show that $\vec{\nabla}.\vec{r} = 3$, where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$.

Contd.

(c) Write the order and degree of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

(d) Write the volume element in curvilinear co-ordinate.

(e) Give the value of $\int_{-\alpha}^{+\alpha}\delta(x)\,dx$

(f) Define variance in statistics.

(g) State the principle of least square fit.

2. Answer of the following questions : 2×4=8

(a) Find a unit vector perpendicular to the surface, $x^2+y^2-z^2=11$ at the point $(4,2,3)$.

(b) If $\vec{A}=\vec{A}(t)$, then show that

$$\frac{d}{dt}\left[\vec{A}.\left(\frac{d\vec{A}}{dt}\times\frac{d^2\vec{A}}{dt^2}\right)\right]=A.\left[\frac{d\vec{A}}{dt}\times\frac{d^3\vec{A}}{dt^3}\right]$$

(c) If $\vec{A}$ and $\vec{B}$ are each irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

(d) Evaluate $\iint_S \vec{r} \times \hat{n} dS$, where S is a closed surface.

3. Answer **any three** of the following questions :

5×3=15

(a) Prove

$$\iiint_V \left(\phi \nabla^2 \psi - \psi \nabla^2 \phi\right) dV = \iint_S \left(\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi\right) . dS$$

(b) Find the integrating factor (IF) of the following differential equation and solve it.

$$\left(1 + x^2\right) \frac{dy}{dx} + 2xy = \cos x$$

(c) Express curl $\vec{A} = \vec{\nabla} \times \vec{A}$ in cylindrical co-ordinate.

(d) What is Dirac-delta function ? Show that the function

$$\delta(x) = \lim_{\varepsilon \to 0} \frac{\sin(2\pi\varepsilon x)}{\pi\varepsilon}$$

is a Dirac delta function.

(e) If $\phi(x, y, z) = 3x^2y - y^3x^2$ be any scalar function ϕ, find out

(i) grad ϕ at point (1, 2, 2)

(ii) unit vector $\hat{e}$ perpendicular to surface.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) If $F_1(x, y), F_2(x, y)$ are two continuous functions having continuous partial derivatives $\frac{\partial F_1}{\partial y}$ and $\frac{\partial F_2}{\partial x}$ over a region R bounded by simple closed curve C in the $x-y$ plane, then show that

$$\oint_C (F_1 dx + F_2 dy) = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\, dy$$

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(ii) A function $f(x)$ is defined

as $\begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \le x \le 4 \\ 0, & x > 2 \end{cases}$

Show that it is a probability density function. 3

(b) Solve the following differential equations : 5+5=10

(i) $9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 6e^{-2x/3}$

(ii) $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$

(c) (i) A rigid body rotates about an axis passing through the origin with angular velocity $\vec{\omega}$ and with linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that,

$$\vec{\omega} = \frac{1}{2}\left(\vec{\nabla} \times \vec{v}\right)$$

where, $\vec{\omega} = \hat{i}\omega_1 + \hat{j}\omega_2 + \hat{k}\omega_3$

$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ 5

(ii) If $y = f(x+at) + g(x-at)$, show that it satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

where f and g are assumed to be at least twice differentiable and a is any constant. 5

(d) (i) Apply Green's theorem in plane to evaluate the integral

$\oint_C \left[\left(xy - x^2 \right) dx + x^2 y \, dy \right]$ over the triangle bounded by the line $y = 0$, $x = 1$ and $y = x$. 6

(ii) Prove that

$$\int_{-\alpha}^{+\alpha} f(x)\, \delta(x-c)\, dx = f(c)$$ 4

(e) (i) Applying Gauss' theorem, evaluate

$$\iint_S x\,dydz + y\,dzdx + z\,dxdy,$$ where

S is the sphere of radius

$x^2 + y^2 + z^2 = 1$ 5

(ii) Evaluate $\nabla^2 \psi$ in spherical co-ordinate. 5

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3 (Sem-1/CBCS) PHY HC 2

2021

(Held in 2022)

PHYSICS

(Honours)

Paper : PHY-HC-1026

(Mechanics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option of the following : $1\times7=7$

(a) Motion of a particle is said to be simple harmonic, if

(i) the motion is along a straight line

(ii) the particle moves back and forth over the same path

(iii) acceleration of the particle is proportional to its displacement

(iv) All of the above.

Contd.

(b) A man pushes a wall but fails to displace it. The work done in this process is

(i) negative work

(ii) positive but less work

(iii) positive and large work

(iv) no work

(c) The moment of inertia of a spherical shell about its diameter is

(i) $\frac{2}{3}MR^2$

(ii) $\frac{2}{5}MR^2$

(iii) $\frac{5}{3}MR^2$

(iv) $\frac{7}{5}MR^2$

(d) If the radius of earth were to shrink by one per cent (its mass remaining the same), then the acceleration due to gravity on the earth's surface

(i) would decrease

(ii) would remain unchanged

(iii) would increase

(iv) Cannot be predicted

(e) Length contraction happens

(i) along perpendicular to the direction of motion

(ii) along the direction of motion

(iii) Both of the above.

(iv) None of the above.

(f) In a rotating frame a body experiences a force in radially outward direction. This is called

(i) centripetal force

(ii) centrifugal force

(iii) Coriolis force

(iv) Newton's force

(g) For a given material, the value of Young's modulus (Y) is 2.4 times of its shear modulus (η). The value of Poisson's ratio will be

(i) 2.4

(ii) 1.2

(iii) 0.4

(iv) 0.2

2. Answer the following questions : 2×4=8

(a) Calculate the angular momentum of the spherical earth rotating about its own axis. [Mass of the earth = $6 \times 10^{24} kg$ and mean radius = $6.4 \times 10^6 m$]

(b) Estimate whether the following force is conservative or not ?

$$\vec{F} = \frac{\alpha}{r^4} x\hat{i} + \frac{\alpha}{r^4} y\hat{j} + \frac{\alpha}{r^4} z\hat{k}$$

(c) A symmetrical body is rotating about its axis of symmetry, its moment of inertia about the axis of rotation being $1 kg m^2$ and its rate of rotation is $2 rev\, s^{-1}$. What is the angular momentum of the body ?

(d) Calculate Poisson's ratio for silver. Given Young's modulus for silver is $7.25 \times 10^{10} Nm^{-2}$ and bulk modulus is $11 \times 10^{10} Nm^{-2}$.

3. Answer ***any three*** of the following : $5\times3=15$

(a) State the work-energy theorem. Prove work-energy theorem for a variable force. $1+4=5$

(b) Write Poiseuille's equation for liquid flow through a narrow tube and also state its corrected form. State the *two* possible sources of error in the Poiseuille's experiment. Define streamline motion. $2+2+1=5$

(c) Point out *two* limitations of Newton's laws of motion. The position vectors of two particles are respectively $\vec{r}_1 = 4\hat{i} + 6\hat{j} + 4\hat{k}$ (in meters) and $\vec{r}_2 = 2\hat{i} + 3\hat{j} + 2\hat{k}$ (in meters). The velocity of the first particle is $\hat{i} - 2\hat{j} + 2\hat{k}$ (in ms^{-1}). What should be the velocity of the second particle so that the two may collide in 5 s. (Here, $\hat{i}$, $\hat{j}$ and $\hat{k}$ denote unit vectors along x, y and z axes, respectively). $2+3=5$

(d) Define inelastic collision. Discuss inelastic collision in the laboratory frame of reference and hence calculate the lost kinetic energy during the collision.

1+4=5

(e) What do you mean by fictitious force in accelerated frames? What is Coriolis force? Under what condition does it come to play? 2+1+2=5

4. Answer ***any three*** of the following:

10×3=30

(a) Establish the equation of motion of a damped harmonic oscillator subjected to resistive force that is proportional to the first power of its velocity. If the damping is less than critical, show that the motion of the system is oscillatory with its amplitude decaying exponentially with time. If the displacement of a moving particle at any time is given by

$$x = a\cos\omega t + b\sin\omega t,$$

find the time period and maximum velocity of the particle, if $a = 6$, $b = 8$ and $\omega = 3$. 4+3+3=10

(b) What do you understand by moment of inertia of a body? Briefly explain the concept of radius of gyration. State and prove the theorem (or principle) of perpendicular axes for a plane laminar body.

For a diatomic molecule in stable equilibrium and having a bond length r_0 (the distance between the two atoms in the diatomic molecule), show that the moment of inertia of the diatomic molecule is given by

Moment of Inertia =(*reduced mass of the molecule*) × (*bond length*)^2

Assume that if m_1 and m_2 are the masses of the atoms of the diatomic molecule, its reduced mass is given by

$$\mu = \frac{m_1 m_2}{m_1 + m_2}.$$

1+1+4+4=10

(c) Give two characteristics of central force. Derive an expression for the reduction of two body problem to one body under central force.

Calculate the period of revolution of Neptune round the Sun, given that the diameter of the orbit is 30 times the diameter of the Earth's orbit round the Sun, both orbits being assumed to be circular. 2+5+3=10

(d) Show that Young's modulus Y; modulus of rigidity η and Poisson's ratio σ are related by the equation $Y = 2\eta(1+\sigma)$.

A rigid rod of $1.5\,m$ in length is fixed horizontally at one end and loaded at the other by a mass of $0.1\,kg$. Calculate the depression of a point distant $1.2\,m$ from the fixed end. Diameter of the rod is 2 cm. Young's modulus of the material of the rod is $1.01 \times 10^{11}\ Nm^{-2}$. 6+4=10

(e) Deduce the mathematical expression for the law of addition of relativistic velocities. Show that in no case the resultant velocity of a material particle can be greater than c. 7+3=10

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