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3 (Sem–1/CBCS) PHY HC 1

2020

(Held in 2021)

PHYSICS

(Honours)

Paper : PHY–HC–1016

(Mathematical Physics–I)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) What is the geometrical interpretation of the scalar triple product of three vectors ?

Contd.

(b) If $\vec{R}(u) = \dfrac{d}{du}\vec{S}(u)$, find $\int_a^b \vec{R}(u)\,du$.

(c) Find the Laplacian of the scaler field $\phi = xy^2z^3$.

(d) Determine the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right) + x^2\left(\frac{dy}{dx}\right)^2 = 0$$

(e) What are the coordinate surfaces in orthogonal curvilinear coordinates ?

(f) Define Dirac delta function.

(g) Write the difference between Systematic error and Random error.

2. Answer ***any four*** of the following questions : 2×4=8

(a) If $\vec{A}(t)$ has a constant magnitude, then show that $\dfrac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

(b) Prove that, the vector $\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is solenoidal.

(c) Show that $\iint_S \vec{A}.\hat{n}\, dS$, over any closed surface S is equal to $\iint_R \vec{A}.\hat{n}\, \dfrac{dx\,dy}{\left|\hat{n}\,.\,\hat{k}\right|}$, where R is the projection of S on xy-plane.

(d) Solve the differential equation

$$xy\,(y+1)\,dy = \left(x^2+1\right)dx\,.$$

(e) State the transformation relation between the spherical polar co-ordinates (r, θ, ϕ) and Cartesian co-ordinates (x, y, z). Obtain the volume elements in spherical polar co-ordinate.

3. Answer ***any three*** of the following questions : 5×3=15

(a) How will you define divergence and curl of a vector $\vec{V}$. Evaluate $\vec{\nabla}.\vec{r}$ and $\vec{\nabla}\times\vec{r}$.

(b) If $\vec{A}$ is a vector, prove that $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}$.

(c) Test the Exactness of the differential equation

$$\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$$

and then solve it.

(d) Express $\nabla^2\psi$ in orthogonal curvilinear coordinates.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Show that the surface integral of a vector $\vec{F}$ and the volume integral of the divergence of the same vector obey the relation :

$$\iint_S \vec{F} \, . \, d\vec{S} = \iiint_V \left(\vec{\nabla} \, . \, \vec{F} \right) dV$$ 6

(ii) Evaluate $\iint_S \vec{r} \, . \, \hat{n} \; dS$, where S is a closed surface. 4

OR

(b) Prove that $\oint_C \vec{A} \, . \, d\vec{\lambda} = \int_S \left(\vec{\nabla} \times \vec{A} \right) . \, d\vec{S}$, where C is the curve bounding the surface S. Hence find $\oint \vec{r} \, . \, d\vec{r}$.

8+2=10

(c) Solve the following differential equations : 5+5=10

(i) $\left(1+x^{2}\right)\frac{dy}{dx}+2xy=\cos x$

(ii) $\frac{d^{2}y}{dx^{2}}-3\frac{dy}{dx}+2y=0$, subject to the condition $y(0)=0$ and $y'(0)=1$.

(d) (i) Prove that spherical polar coordinate system is orthogonal. 6

(ii) The probability density function of a variable X is

X :	0	1	2	3	4	5	6
$P(X)$:	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find $P(X<4)$, $P(X\geq 5)$, $P(3<X\leq 6)$. Here $P(X)$ is a probability density function. 4

(e) (i) Prove the expression

$\int_{-\infty}^{+\infty} \delta(x)dx = 1$ where $\delta(x) = 0$ if

$x \neq 0$ and $\delta(x) = \infty$ if $x = 0$. 5

(ii) Given the three vectors

$$\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$$
$$\vec{B} = \hat{j} + \hat{k}$$
$$\vec{C} = \hat{i} - \hat{j}$$

Evaluate $\vec{A} \times (\vec{B} \times \vec{C})$ and show that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$$

2+3=5

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3 (Sem–1/CBCS) PHY HC 2

2020

(Held in 2021)

PHYSICS

(Honours)

Paper : PHY–HC–1026

(Mechanics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct option of ***any seven*** of the following : $1 \times 7 = 7$

   (a) A rocket is based on the principle of conservation of :

   (i) Linear momentum

   (ii) Angular momentum

   (iii) Energy

   (iv) Mass

Contd.

(b) The displacement of a particle is represented by the equation $y = A sin\omega t + B cos\omega t$. The motion of particle is :

(i) Simple harmonic motion with amplitude A

(ii) Oscillatory but not simple harmonic

(iii) Simple harmonic motion with amplitude $A+B$

(iv) Simple harmonic motion with amplitude $\sqrt{(A^2+B^2)}$

(c) Which one of the following is not a unit of Young's modulus ?

(i) Nm^{-1}

(ii) Nm^{-2}

(iii) $dyne\ cm^{-2}$

(iv) *megapascal*

(d) If the moment of inertia of a solid sphere of mass M and radius R about its diameter is $\frac{2}{5}MR^2$, it's moment of inertia about a tangent is given by :

(i) $\frac{5}{7}MR^2$

(ii) $\frac{2}{3}MR^2$

(iii) $\frac{7}{5}MR^2$

(iv) $\frac{3}{5}MR^2$

(e) Which of the following relation between gravitational force $\vec{F}$ and potential V is correct ?

(i) $\vec{F} = -grad\ V$

(ii) $\vec{F} = -div\ V$

(iii) $\vec{F} = -curl\ V$

(iv) $\vec{F} = -\int V\,dx$

(f) Mathematical formulation $E = pc$ applies for :

(i) Electron

(ii) Photon

(iii) A massless particle moving fast

(iv) A particle moving with speed of light

(g) A particle is at rest in a rotating frame. The pseudo force acting on the particle in the rotating frame is :

(i) Zero

(ii) Only the centrifugal force

(iii) Only the coriolis force

(iv) The combination of both centrifugal and coriolis force

(h) The velocity profile of a liquid flowing in a capillary tube of uniform cross-section is :

(i) Circular

(ii) Elliptical

(iii) Parabolic

(iv) Hyperbolic

2. Answer ***any four*** of the following : 2×4=8

(a) A loop of mass 2 *kg* and radius 20 *cm* rolls along the ground at the rate of 10 ms^{-1} . Calculate its kinetic energy.

(b) In one-dimensional motion of a mass of 10 *gm*, it is acted by a restoring force of 10 *dyne* cm^{-1}, and a resisting force of magnitude 2 *dynes-sec* cm^{-1}. Find the value of resisting force which will make the motion critically damped.

(c) At what velocity will a 10,000 *kg* truck have the same momentum as a 4000 *kg* car at 30 ms^{-1} ?

(d) In case of a damped harmonic oscillator, calculate the time in which its amplitude falls to e^{-1} of its undamped value. (Given, $k = \frac{1}{2\tau} = 0 \cdot 02$).

(e) Find out whether the following force is conservative or not.

$$\vec{F} = \left(2xy + z^2\right)\hat{i} + x^2\,\hat{j} + 2xz\,\hat{k}$$

3. Answer ***any three*** of the following :

(a) Prove that in a conservative field force is negative gradient of potential. 5

(b) What is Coriolis force ? Obtain an expression for the horizontal displacement of an object falling freely under the action of gravity in latitude φ. 1+4=5

(c) Define centre of mass in terms of linear momentum of a system. Show that Newton's second law of motion remains invariant under Galilean transformation. 1+4=5

(d) What is a central force ? Prove that the path of a particle under a central force lie in a plane. 1+4=5

(e) What do you mean by laboratory frame of reference ? Show that, in the center-of-mass frame of reference, the magnitude of the velocities of the particles remain unaltered in an elastic collision. 1+4=5

4. Answer ***any three*** of the following :

(a) What are transient and steady states of a forced harmonic oscillator ? Establish the differential equation for a forced harmonic oscillator and find its steady state solution. 3+3+4=10

(b) Find the moment of inertia of a rectangular plate with sides 'a' and 'b' about a side. Derive an expression for the acceleration of a body rolling down an inclined plane. 5+5=10

(c) Obtain an expression for gravitational potential at a point outside a hollow spherical shell. Draw a diagram to represent the variation of gravitational potential of a spherical shell against distance (r) from its centre. The radius of the earth is $6{\cdot}37 \times 10^{8}\ cm$, its mean density, $5{\cdot}5\ gm\,cm^{-3}$ and the gravitational constant, $6{\cdot}66 \times 10^{-8}$ CGS units. Calculate the earth's surface potential.

5+2+3=10

(d) A uniform beam is clamped at one end and loaded at the other. Obtain the relation between the load and depression at the loaded end when the weight of the beam can be neglected. Young's modulus of a substance is equal to 7×10^{11} *dynes* cm^{-2}, and the rigidity modulus for the same substance is equal to 3×10^{11} *dynes* cm^{-2}. Calculate the bulk modulus of elasticity and Poisson's ratio for the substance. 6+4=10

(e) On the basis of Lorentz transformation, discuss *(i)* length contraction and *(ii)* simultaneity of two events. How fast a spacecraft need to travel relative to the earth for each day on the spacecraft to correspond to two days on the earth ? 3+3+4=10