Total number of printed pages–11

2022

STATISTICS

(Honours)

Paper : STA–HC–1016

(Descriptive Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : *(any seven)* $1 \times 7 = 7$

   (a) The point of intersection of the 'less than type' and 'more than type' ogive corresponds to ________. *(Fill in the blank)*

   (b) What do you mean by price relative ?

(c) If $x_i/f_i\,(i=1,2,...,n)$ is a frequency distribution and $u_i=\frac{x_i-a}{h}$, then which one of the following is true

(i) $\overline{x}=\overline{u}$

(ii) $\overline{u}=h\overline{x}$

(iii) $\overline{x}-a=h\overline{u}$

(iv) $\overline{u}=a\overline{x}+h$

Symbols have their usual meaning.

(d) Define ordinal data.

(e) The value of mean deviation is minimum when deviations are taken w.r.t. ________. *(Fill in the blank)*

(f) The signs of the two regression coefficients are different. *(State true or false)*

(g) For an asymmetrical distribution mean = 5, median = 4. Find the value of mode.

(h) Write Sheppard's correction for μ_4.

(i) Give the definition of variance for a frequency distribution $x_i/f_i\,(i=1,2,...,n)$.

(j) State the advantage of coefficient of variation over standard deviation.

(k) Write *two* demerits of geometric mean.

(l) If one of the regression coefficient is 1, the other must be

(i) greater than 1

(ii) lie between −1 and zero

(iii) less than 1

(iv) lie between −1 to +1

(Choose the correct option)

2. Answer ***any four*** of the following questions : 2×4=8

(a) Mention *two* limitations of statistics.

(b) Distinguish between frequency and non-frequency data.

(c) Define multiple correlation and partial correlation for a distribution involving the variables X_1, X_2 and X_3.

(d) What do you mean by dichotomous and manifold classification of attributes ?

(e) Prove that Fisher's index number satisfies factor reversal test.

(f) For what value of A the quantity

$$\sum_{i=1}^{n} f_i (x_i - A)^2$$

would be minimum ? Prove that.

(g) Define absolute moments and factorial moments.

(h) Which is the best measure of dispersion and why ?

3. Answer ***any three*** questions : 5×3=15

(a) Define raw moments and central moments. Derive the relationship between nth central moment and raw moments about the origin. 2+3=5

(b) Explain a histogram. How would you draw a histogram when the width of all classes are not equal ? State how a histogram is different from a bar diagram. 2+2+1=5

(c) Give *two* values x_1 and x_2, prove that

$$AM \geq GM \geq HM$$

Also show that

$$HM = (GM)^2 / AM$$ 3+2=5

(d) (i) Prove that A.M. of the two regression coefficients is greater than the correlation coefficient.

(ii) Examine the consistency of the following data :

$$N = 1000, (A) = 600, (B) = 500, (AB) = 50$$

(iii) When two attributes are said to be positively associated ?

2+2+1=5

(e) (i) Define CLIN. Interprete the result CLIN = 130.50

(ii) Mention *two* sources of secondary data.

(iii) What is a box plot ? 2+1+2=5

(f) How would you determine median graphically by using

(i) single ogive

(ii) both the ogives ? 2+3=5

(g) Write a note on skewness and kurtosis including different measures for them and relevant diagrams.

(h) Find the mean deviation from the mean and standard deviation of A.P. $a, a+d, a+2d, \ldots, a+2nd$ and verify that the latter is greater than the former. 4+1=5

4. Answer ***any three*** questions : 10×3=30

(a) (i) Prove that $-1 \le r_{XY} \le +1$ 3

(ii) Are two uncorrelated variables essentially independent. If not, prove it with the help of an example. 2

(iii) Discuss the steps involved in the construction of wholesale price index numbers. 5

(b) (i) If for a random variable X the absolute moment of order k exists for ordinary $k = 1, 2, \ldots, n-1$, then the following quantities :

$$\beta_k^{2k} \le \beta_{k-1}^k \cdot \beta_{k+1}^k \text{ and}$$

$$\beta_k^{1/k} \le \beta_{k+1}^{1/k+1}$$

hold for $k = 1, 2, \ldots, n-1$, where β_k is the k^{th} absolute moment about the origin. 4+1=5

(ii) Show that in a discrete series its deviations are small compared with mean μ so that $(x/M)^3$ and higher power of (x/M) are neglected, we have

$$G = M\left(1 - \frac{1}{2}\frac{\sigma^2}{M}\right)$$

where M is the arithmetic mean and G is the geometric mean. 5

(c) (i) Define the measures of association Q and Y and show that

$$Q = \frac{2Y}{1+Y^2}$$ 2+3=5

(ii) Write the properties of multiple correlation coefficient. What is the significance of partical correlation coefficient in regression analysis ? 3+2=5

(d) (i) Describe the term 'deflation' in index number. 2

(ii) Find the angle between two lines of regression and interprete the result for $r = 0$ and $r = \pm 1$. 4+1=5

(iii) The regression equation of x on y is

$$3y - 5x + 180 = 0$$

Given that $\overline{y} = 4$, $\sigma_x^2 = \frac{9}{16}$ and $n = 4$. Find r and $\overline{x}$. 3

(e) (i) Show that Laspeyre's and Paasche's index numbers do not satisfy the time and factor reversal tests of consistency. 5

(ii) Interpret the meaning of the statement

$$byx = -0.53$$ 1

(iii) Distinguish between observational studies and controlled experiment with example. 4

(f) (i) Discuss the method of least squares for fitting a straight line $Y = a + bx$. 5

(ii) Find the regression line of Y on X. At which point this line intersects the regression line of X on Y. 4+1=5

(g) (i) Write a note on Sheppard's correction for moments. Define Pearson's β and γ coefficient. 3+2=5

(ii) Show that in usual notation

$$1 - R_{1.23}^2 = \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right)$$ 3

(iii) Define partial correlation coefficient. 2

(h) (i) Write a note on scrutiny of data for internal consistency and detection of errors. 5

(ii) Give idea of cross-validation. 2

(iii) Write a note on chain index numbers. 3

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3 (Sem–1/CBCS) STA HC 2

2022

STATISTICS

(Honours)

Paper : STA–HC–1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : ***(any ten)*** $1\times10=10$

   (a) If a function is derivable at all points of an interval except the ends points, it is said to be derivable in the open interval. *(State True* ***or*** *False)*

   (b) The value of the integral

   $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ is

   (i) $\beta(m,n)$

Contd.

(ii) $\beta(n,m)$

(iii) Both (i) and (ii)

(iv) None of the above

(Choose the incorrect option)

(c) The nth derivative of a^x is

(i) a^x

(ii) $(\log_e a)^n a^x$

(iii) na^x

(iv) None of the above

(Choose the correct option)

(d) Evaluate $\int_0^\infty e^{-3x} x^{\frac{1}{2}} dx$

(e) The differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 - 2\left(\frac{dy}{dx}\right)^2 + 5y = 0$$ is of order

_____ and degree _____.

(Fill in the blanks)

(f) State *two* properties of definite integrals.

(g) Define homogeneous function of two variables.

(h) If $f(x,y) = x^4 + xy + y^4$ find f_x and f_{yx}.

(i) The value of

$$\lim_{x\to\infty} \frac{x^2}{e^x}$$ is

(i) ∞

(ii) 0

(iii) $\frac{\infty}{\infty}$

(iv) None of the above

(Choose the correct option)

(j) Write *two* properties of double integrals.

(k) The differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy$$ is called

(i) ordinary differential equation

(ii) partial differential equation

(iii) None of the above

(Choose the correct option)

(l) Find the differential equation of all the straight lines passing through the origin.

(m) Define beta integral of second kind.

(n) Lagrange's undetermined multipliers is a method of finding the _______ or _______ of a function subject to one or more conditions. *(Fill in the blanks)*

(o) Define Jcobian of the functions $u_1, u_2 \ldots u_n$ with respect to $x_1, x_2 \ldots x_n$.

(p) The value of $\Gamma(n+1)$ is

(i) $n!$

(ii) $n\Gamma(n)$

(iii) Both (i) and (ii)

(iv) $(n-1)!$

(Choose the incorrect option)

(q) The function x^n is continuous for all values of x when n is positive and continuous for all values of x except 0 when n is negative. *(State True or False)*

(r) Define bounded function.

2. Answer **any five** of the following questions : 2×5=10

(a) Test the differentiability of the function

$$f(x) = \begin{cases} 1+x, \text{ if } x \le 2 \\ 5-x, \text{ if } x > 2 \end{cases}$$

at $x = 2$

(b) Find the nth differential coefficients of $\sin^3 x$.

(c) If $f(x) = x.\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}, x \neq 0$ and $f(0) = 0$, show that $f(x)$ is continuous at $x = 0$.

(d) Find the value of

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

(e) Show that $f(x) = 2x^3 - 21x^2 + 36x - 20$ has a maximum at $x = 1$.

(f) Prove that

$$\beta(m,n) = \beta(m+1, n) + \beta(m, n+1)$$

(g) Solve the

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$$

(h) Prove that

$$\int_0^{\pi/2} \sqrt{tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$$

(i) If $x^3 + 3x^2y + 6xy^2 + y^3 = 1$, find $\frac{dy}{dx}$.

(j) If u be a homogeneous function of x and y of degree n, then show that

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x}$$

3. Answer **any four** from the following questions : 5×4=20

(a) Show that the function $f(x) = |x| + |x-1|$ is not differentiable at $x = 1$ but differentiable at $n = 2$.

(b) If $y = e^{a\sin^{-1}x}$, prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$

(c) If $u = log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

(d) Find the solution

$$\lim_{x \to 0} \frac{tan\, x - x}{x - sin\, x}$$

(e) If f_x and f_y are both differentiable at a point (a,b) of domain of definition of a function f, then

$$f_{xy}(a,b) = f_{yx}(a,b)$$

(f) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$$

(g) If $0 < n < 1$, then show that

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{sin\, n\pi}$$

Hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(h) Evaluate

$$\int_0^{\pi/2}\int_{\pi/2}^{\pi} cos\,(x+y)dy\, dx$$

4. Answer **any four** from the following questions : 10×4=40

(a) (i) Prove that

$$\int_{-a}^{a} f(x)dx = 0, \text{ if } f(x) \text{ is and odd function of } x.$$

$$= 2\int_{0}^{a} f(x)dx, \text{ if } f(x) \text{ is an even function of } (x).$$ 3

(ii) Using properties of definite integral prove that

$$\int_{0}^{\pi/2} \log \sin x \, dx = \frac{\pi}{2} \log \frac{1}{2}$$ 7

(b) (i) Find the differential coefficient of

$$x^x + (\sin x)^{\log x}$$ 5

(ii) If $y = e^x \log x$, show that in usual notation

$$xy_2 - (2x-1)y_1 + (x-1)y = 0$$ 5

(c) (i) If $f(x,y) = \frac{2xy(x^2 - y^2)}{x^2 + y^2}$,

$(x,y) \neq (0,0)$

$f(0,0) = 0$, find $f_x(0,0)$ and $f_y(0,0)$ 5

(ii) If $u = \log\left\{\frac{x^2 + y^2}{x + y}\right\}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$$ 5

(d) (i) Explain general solution of Clairaut's equation. 4

(ii) Solve the equation $(px - y)(x - yp) = 2p$ to Clairaut's form by the substitution $x^2 = u$, $y^2 = v$, and find its solution. 6

(e) (i) Show that

$$\int_{0}^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_{0}^{\pi/2} \sqrt{\sin x}\, dx = \pi$$ 6

(ii) Prove that

$$\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$$ 4

(f) (i) Explain the procedure of equations solvable for p and y. 5

(ii) Solve the differential equation

$$\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - 56 = 0$$ 5

(g) (i) State the necessary and sufficient condition for extreme value of a function of two variables. 4

(ii) Find the maximum value of

$$f(x,y) = 3x^2 - y^2 + x^3$$ 6

(h) Prove that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx$$

(i) The roots of the equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ are u, v, w. Prove that

$$\frac{\partial(u\,v\,w)}{\partial(x,y,z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

(j) (i) Define partial differential equation. 1

(ii) Solve the partial differential equations

(A) $\left(\frac{y^2 z}{x}\right) p + xzq = y^2$ 5

(B) $xp + yq = z$ 4
