Total number of printed pages–7

3 (Sem–1/CBCS) STA HC 1

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA–HC–1016

(Descriptive Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

   (a) The column headings of a statistical table are known as

   (i) sub-titles

   (ii) stubs

Contd.

(iii) reference notes

(iv) captions

(Choose the correct option)

(b) If 5 is subtracted from each observation of a set, then the arithmetic mean of the new set of observations is reduced by ______.

(Fill in the blank)

(c) The best measure of dispersion for comparison of *two* different series is coefficient of variation.

(State True or False)

(d) With usual notations, if for two attributes A and B, $(AB) > \frac{(A)(B)}{N}$, the attributes are

(i) independent

(ii) positively associated

(iii) negatively associated

(iv) None of the above

(Choose the correct option)

(e) Laspeyres price index number uses the ________ quantities as weights.

(Fill in the blank)

(f) If X and Y are independent, the value of regression coefficient β_{YX} is equal to

(i) 1

(ii) ∞

(iii) 0

(iv) None of the above

(Choose the correct option)

(g) The partial correlation coefficient lies between $-\infty$ and $+\infty$.

*(State True **or** False)*

2. Answer the following questions : 2×4=8

(a) State *two* limitations of statistics.

(b) For a distribution, mean is 10 and variance is 16. Find the first two moments about origin.

(c) Prove that Paasche's index number does not satisfy the time reversal test.

(d) "The regression coefficient of X on Y is 3.2 and that of Y on X is 0.8." Is this statement correct ? Give reasons in support of your answer.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Give a brief description of different components of a statistical table. 5

(b) What is standard deviation ? Find standard deviation of the first n natural numbers. 1+4=5

(c) Define multiple and partial correlation coefficient. If $r_{12} = 0.85, r_{13} = 0.65$ and $r_{23} = 0.72$; find $R_{1.23}$. (Notations having usual meaning.) 2+3=5

(d) Suppose P_{01}^{La}, P_{01}^{Pa} and P_{01}^{ME} denote Laspeyres, Paasche and Marshall-Edgeworth price index numbers respectively. If $P_{01}^{La} < P_{01}^{Pa}$, then prove that

$$P_{01}^{La} < P_{01}^{ME} < P_{01}^{Pa}$$ 5

(e) Obtain the normal equations for fitting of the 2^{nd} degree parabola $y = a + bx + cx^2$ on the basis of n pairs $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$ of values of (X, Y). 5

4. Answer either (a) or (b) : 10

(a) (i) Distinguish between attributes and variables. 2

(ii) Discuss the construction of cost of living index number by family budget enquiry. 3

(iii) Prove that correlation coefficient lies between -1 and $+1$. Give the geometrical interpretation of the case when $r = +1$. 4+1=5

(b) (i) Write a brief note on consistency of data with special reference to attributes. 2

(ii) Write a note on selection of base period in construction of index number. 3

(iii) Prove that regression coefficients are independent of charge of origin but not of scale. 5

5. Answer either *(a)* **or** *(b)* : 10

(a) *(i)* Write briefly on control experiments. 2

(ii) Find the arithmetic mean of the AP series a, $a+d$, $a+2d, \ldots\ldots$, $a+2nd$. 3

(iii) Elaborate on the uses of cost of living index number. 5

(b) *(i)* What does Karl Pearson correlation coefficient measure ? 1

(ii) Define mode and derive its formula. 1+5=6

(iii) State the properties of multiple correlation coefficient. 3

6. Answer either *(a)* **or** *(b)* : 10

(a) *(i)* State the values of β_1 and β_2 for a symmetric distribution. 1

(ii) Write a brief note on box plot. 3

(iii) Derive the formula for Spearman's rank correlation coefficient in case of non-repeated ranks. 6

(b) *(i)* Define chain-based index number. 2

(ii) What is skewness ? State various measures of skewness. 1+2=3

(iii) With usual notations, prove that

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1 - r_{13}^2)(1 - r_{23}^2)}} \qquad 5$$

Total number of printed pages–8

3 (Sem–1 /CBCS) STA HC 2

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA–HC–1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×10=10

(a) Define differential coefficient of $f(x)$ at the point $x = a$.

(b) The value of $\lim_{x \to 0} \frac{\tan x}{x}$ is

(i) 0

(ii) 1

(iii) α

(iv) None of the above

(Choose the correct option)

Contd.

(c) Evaluate $\Gamma\left(-\frac{3}{2}\right)$.

(d) State Leibnitz's theorem.

(e) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(f) Find the differential equation of lines parallel to x-axis.

(g) The integral $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$ converges if

(i) $m > 0, n > 0$

(ii) $m < 0, n > 0$

(iii) $m > -1, n > -1$

(Choose the correct option)

(h) If $f(x,y) = 2x^2 - xy + 2y^2$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1,2)$.

(i) The differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 - 2\left(\frac{dy}{dx}\right)^2 + 5y = 0 \quad \text{is}$$

(i) an ordinary differential equation

(ii) of order two and degree two

(iii) called partial differential equation

(Choose the incorrect option)

(j) Find the value of

$$\lim_{x \to \alpha} \frac{x^4}{e^x}$$

2. Answer the following questions : 2×5=10

(a) Examine the differentiability at $x=0$ of the function f defined on the set of real number as follows :

$$f(x) = x^2 \sin\frac{1}{x}, \text{ if } x \neq 0$$
$$= 0, \text{ if } x = 0$$

(b) Evaluate $\lim_{x \to 0} (\sin x \log x)$

(c) Show that $f(x) = x^3 - 6x^2 + 24x + 1$ has neither a maximum nor a minimum.

(d) Obtain a differential equation from the relation

$$y = A\sin x + B\cos x + x\sin x$$

(e) Show that for $l > 0,\ m > 0$

$$\int_a^b (x-a)^{l-1}(b-x)^{m-1}\,dx = (b-a)^{l+m-1}\,\beta(l,m)$$

3. Answer ***any four*** from the following questions : 5×4=20

(a) Show that if a function is differentiable at a point, then it is continuous at that point but the converse is not necessarily true.

(b) Show that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be be exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(c) Evaluate $\int_1^{\log 8}\int_1^{\log y} e^{x+y}\,dy\,dx$

(d) If (a,b) be a point of the domain of definition of a function f such that

(i) f_x is continuous at (a,b)

(ii) f_y exists at (a,b), then show f is differentiable at (a,b).

(e) If $u = \sin^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then using Euler's theorem show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$$

(f) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

4. (a) (i) If $y = \sin^{-1}x$, then using Leibnitz's theorem prove that

$$\left(1-x^2\right)y_{n+2} - (2n+1)x\,y_{n+1} - n^2 y_n = 0$$

6

(ii) Test the continuity and differentiability of the function

$$f(x) = \begin{cases} 1+x & \text{if} \quad x \le 2 \\ 5-x & \text{if} \quad x \ge 2 \end{cases}$$ 4

at $x=2$

Or

(b) Solve the differential equation

$$\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$$ 10

5. (a) (i) For a positive number P, show that

$$\Gamma(P)\,\Gamma\left(P+\frac{1}{2}\right)2^{2P-1} = \sqrt{\pi}\,\Gamma(2P)$$ 6

(ii) Evaluate $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$ 4

Or

(b) (i) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ 5

(ii) If $u = 2(ax+by)^2 - (x^2+y^2)$ and $a^2+b^2=1$, find the value of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}.$$ 5

6. (a) (i) Show that the function $u = x^3 + y^3 - 3ay$ has a maximum or minimum at the point (a, a) according as a is negative or positive. 5

(ii) If $f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$; $(x,y) \neq (0,0)$, $f(0,0)=0$, then show that at the origin $f_{xy} \neq f_{yx}$. 5

Or

(b) (i) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = \sin x$$ 5

(ii) Define Clairaut's equation. Explain the general solution of Clairaut's equation. 5

7. (a) (i) If $u^3+v^3=x+y$,

$u^2+v^2=x^3+y^3$, prove that

$$\frac{\partial(u,v)}{\partial(x,y)}=\frac{y^2-x^2}{2uv(u-v)}$$ 5

(ii) Solve the partial differential equation : 5

$$\left(\frac{y^2z}{x}\right)P+xzq=y^2$$

Or

(b) If f is defined and continuous on the rectangle $R=[a,b;c,d]$, and if

(i) $f_x(x,y)$ exists and is continuous on the rectangle R, and

(ii) $g(x)=\int_c^d f(x,y)dy$ for $x\in[a,b]$

then show that g is differentiable on $[a,b]$ and $g'(x)=\int_c^d f_x(x,y)dy$

10