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3 (Sem–1/CBCS) STA HC 1

2020

(Held in 2021)

STATISTICS

(Honours)

Paper : STA-HC-1016

(Descriptive Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1\times7=7$

(a) It is necessary to find cumulative frequencies in order to draw a/an

(i) histogram

(ii) frequency polygon

(iii) ogive

(iv) pie chart

(Choose the correct option)

Contd.

(b) If the harmonic mean of the two numbers 'a' and 'b' is 5 and if $a = 5$, then b is ______. *(Fill in the blank)*

(c) "Two series A and B have the same standard deviations, but the mean of A is greater than that of B. The coefficient of variation of A is less than that of B". *(State True **or** False)*

(d) For consumer price index, price quotations are collected from

(i) wholesale dealers

(ii) retailers

(iii) fair price shops

(iv) government depots.

(Choose the correct option)

(e) What do you mean by controlled experiment?

(f) In a regression line of Y on X, the variable X is known as

(i) independent variable

(ii) regressor

(iii) explanatory variable

(iv) All of the above.

(Choose the correct option)

(g) State the limits for Spearman's rank correlation coefficient.

2. Answer the following questions : 2×4=8

(a) State with suitable example, the distinction between an attribute and a variable.

(b) Prove that the arithmetic mean of a variable whose given values are all equal, must be the same as their common value.

(c) State *any two* assumptions of Karl Pearson's correlation coefficient.

(d) Give the interpretation of Wholesale price index and Cost of living index number.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Differentiate between—

(i) primary data and secondary data

(ii) questionnaire and schedule.

2½+2½=5

(b) Define absolute moment and factorial moment. If $Y = a + bX$ be a linear function of X ; then prove that the arithmetic means of Y and X are related in the same way as Y and X themselves are. 2+3=5

(c) Write a note on different scales of measurement — nominal, ordinal, interval and ratio.

(d) Explain briefly different types of errors in Index number.

(e) Define Skewness and Kurtosis. For discrete distribution, prove that $\beta_2 > 1$, notation having usual meaning. 2+3=5

4. Answer **either** (a) **or** (b) :

(a) (i) The sum of 10 items is 12 and sum of their squares is 16.9. What is the value of the standard deviation ? 2

(ii) Write a brief note on Sheppard's Correction for moments. 3

(iii) The variables X and Y are connected by the equation $aX + bY + c = 0$. Show that the correlation between them is -1 if the signs of 'a' and 'b' are alike and $+1$ if they are different. 5

(b) (i) Why do we calculate in general, only the first four moments about mean of a distribution and not the higher moments ? 2

(ii) Examine the consistency of the following data —

$N = 1000$; $(A) = 600$; $(B) = 500$; $(AB) = 50$, the symbols having their usual meaning. 2

(iii) If $L(p)$ and $P(q)$ represent respectively Laspeyre's index number for prices and Paasche's index number for quantities, show that

$$\frac{L(p)}{L(q)} = \frac{P(p)}{P(q)}$$ 6

5. Answer **either** (a) **or** (b) :

(a) (i) Differentiate between population and sample. 2

(ii) Define raw and central moments of a frequency distribution. Obtain the relationship between the central moments of order r in terms of the raw moments. 1+4=5

(iii) Briefly describe the term 'deflation' in Index number. 3

(b) (i) Give an idea of scrutiny of data for internal consistency. 2

(ii) For a trivariate distribution, explain partial correlation coefficient with example. 2

(iii) What do you mean by method of least squares? Derive the equation of the line of regression of Y on X. 1+5=6

6. Answer **either** (a) **or** (b) :

(a) (i) Define standard deviation. If n_1, n_2 are the sizes, $\bar{x}_1, \bar{x}_2$, the means and σ_1, σ_2, the standard deviations of two series respectively, then find the standard deviation σ of the combined series of size $n_1 + n_2$. 1+6=7

(ii) Write a note on Index of industrial production. 3

(b) (i) Write a note on choice of weights in construction of index number. 3

(ii) Define Multiple correlation coefficient with usual notations, prove that

$$R^2_{1\cdot23} = \frac{r^2_{12} + r^2_{13} - 2r_{12}r_{13}r_{23}}{1 - r^2_{23}}$$ 1+6=7

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3 (Sem–1/CBCS) STA HC 2

2020

(Held in 2021)

STATISTICS

(Honours)

Paper : STA-HC-1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the questions as directed : 1×10=10

   (a) The value of $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}}$ is

   (i) 0

   (ii) 1

   (iii) None of the above

   (Choose the correct option)

(b) The general solution of the linear differential equation

$$\frac{d^2y}{dx^2} + p_1 \frac{dy}{dx} + p_2 y = 0 \text{ has two}$$

equal roots, then complementary function is

$$y = (c_1 + c_2 x) e^{\alpha x}$$

*(State True **or** False)*

(c) What is the relation between gamma and beta function ?

(d) The value of $D^n e^{ax}$ is

(i) $a^n e^{ax}$

(ii) $\left(e^{ax}\right)^n$

(iii) $n e^{ax}$

(Choose the correct option)

(e) The stationary point which are not extreme points are called __________.

(Fill in the blank)

(f) State Euler's theorem on homogeneous function.

(g) If $f(x) = x^4 + x^2y^2 + y^4$, find f_x, f_{yx}.

(h) The differential equation

$$f(x,y)\left(\frac{d^m y}{dx^n}\right)^p + \phi(x,y)\left(\frac{d^{m-1} y}{dx^{m-1}}\right) + \dots = 0$$

is of order _______ and degree _______.

(Fill in the blanks)

(i) Define integrating factor.

(j) Evaluate —

$$\lim_{x \to 0} \frac{a^x - b^x}{x}$$

2. Answer the following questions : 2×5=10

(a) Prove that

$$\beta(n,n) = \frac{\sqrt{\pi}\,\Gamma(n)}{2^{2n-1}\,\Gamma\left(n+\frac{1}{2}\right)}$$

(b) If $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2} & , \quad (x, y) \neq (0,0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$

show that both the partial derivatives exist at (0,0) but the function is not continuous thereat.

(c) Find the maximum and minimum value of $f(x) = a\sin^2 x + b\cos^2 x$.

(d) Solve $x(y^2+1)dx + y(x^2+1)dy = 0$.

(e) Obtain the differential equation of the family of curves represented by

$y = e^x (A\cos x + B\sin x)$

where A and B are arbitrary constants.

3. Answer ***any four*** from the following questions : 5×4=20

(a) Evaluate

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

(b) By using the transformation $x+y=u,\ y=uv$, show that

$$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}}\, dx\, dy = \frac{1}{2}(e-1)$$

(c) Reduce the equation

$(px-y)(x-py) = 2p$ to Clairaut's form by the transformation $x^2 = u,\ y^2 = v$ and find its complete solution.

(d) If $u = log \dfrac{x^4 + y^4}{x+y}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$$

(e) If $u_1 = \dfrac{x_2 x_3}{x_1},\ u_2 = \dfrac{x_3 x_1}{x_2},\ u_3 = \dfrac{x_1 x_2}{x_3}$

prove that $J(u_1, u_2, u_3) = 4$.

(f) Solve the differential equation

$$(1+y^2)dx = (tan^{-1} y - x)dy$$

4. Answer ***any four*** of the following questions : 10×4=40

(a) (i) Consider the function

$$f(x)=(x-a)\,sin\left(\frac{1}{x-a}\right);\ x\neq 0$$
$$=0 \qquad ;\ x=a$$

Show that $f(x)$ is continuous but not derivable at $x=a$. 6

(ii) Prove that 4

$$\Gamma\left(\frac{3}{2}+x\right)\Gamma\left(\frac{3}{2}-x\right)=\left(\frac{1}{4}-x^2\right)\pi sec\,\pi x,$$
$$-1<2x<1$$

(b) (i) Find the integrating factor of differential equation and solve

$$\left(1+x^2\right)\frac{dy}{dx}+2xy=cos\,x$$ 5

(ii) If $u=f\,(y-z,\ z-x,\ x-y)$, prove that 5

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

(c) (i) Evaluate $\iint (x+y)^2 \, dx\, dy$ over the area bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 .$$ 8

(ii) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta} \, d\theta = \frac{\pi}{\sqrt{2}}$. 2

(d) The roots of the equation

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ are u, v, w. Prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2 \frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

10

(e) (i) Express $\int_0^{\pi/2} \sin^4\theta \cos^6\theta \, d\theta$ as a beta function and hence evaluate it. 2+3=5

(ii) Solve

$y = 3x + \log p$ 5

(f) (i) Dicuss the derivability of the following function

$$f(x) = 2x - 3\ ;\ 0 \le x \le 2$$
$$= x^2 - 3\ ;\ 2 < x \le 4$$

at the point $x=4$. 5

(ii) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = sin 2x$ 5

(g) (i) Solve the partial differential equation $x^2p + y^2p = z^2$ 5

(ii) Show that $\int_0^1 \frac{dx}{\left(1-x^6\right)^{\frac{1}{6}}} = \frac{\pi}{3}$ 5

(h) Show that the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $8abc / 3\sqrt{3}$. 10
