

2022

MATHEMATICS

(Major)

Paper : 1.1

(Algebra and Trigonometry)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following as directed : $1 \times 10 = 10$

(a) What is the condition that union of two subgroups of a group is again a subgroup of the group?

(b) What is the order of element

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 4 & 2 & 5 & 1 & 7 & 8 & 9 \end{pmatrix}$$

of the permutation group P_9 ?

- (c) Is every subgroup of an Abelian group is normal?
- (d) If I_n be a unit matrix of order n , then what is the matrix $\text{adj } I_n$?
- (e) What is the normal form of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$?
- (f) If the non-singular matrix A is symmetric, then
- A is Hermitian
 - A is skew-Hermitian
 - A^{-1} is symmetric
 - A^{-1} is skew-symmetric
- (Choose the correct answer)
- (g) What is the rank of a non-singular matrix of order 3×3 ?
- (h) Express the complex number $-1+i$ in its polar form.
- (i) What is the relation between circular and hyperbolic functions of sine?
- (j) What is the value of $\log_e i$?

2. Answer the following questions : $2 \times 5 = 10$

- (a) If a is a generator of a cyclic group G , then show that a^{-1} is also a generator of G .
- (b) If A is a symmetric matrix, then prove that $\text{adj } A$ is also symmetric.
- (c) With an example, show that a matrix which is skew-symmetric is not skew-Hermitian.
- (d) If A and B be two equivalent matrices, then show that $\text{rank } A = \text{rank } B$.
- (e) If $x + \frac{1}{x} = 2 \cos \theta$, then show that

$$x^n + \frac{1}{x^n} = 2 \cos n\theta$$

3. Answer the following questions : $5 \times 2 = 10$

- (a) If H is a subgroup of a group G and N is a normal subgroup of G , then show that $H \cap N$ is a normal subgroup of H .

- (b) Prove that n , n th roots of unity forms a series in GP.

Or

Show that

$$1 - \frac{2}{\sqrt{3}} + \frac{3}{\sqrt{5}} - \frac{4}{\sqrt{7}} + \dots \infty = \frac{1}{\sqrt{2}} \sin\left(\frac{\pi}{4} + 1\right)$$

4. Answer any two questions : $5 \times 2 = 10$

- (a) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then form an equation whose roots be $(\alpha - \beta)^2, (\beta - \gamma)^2, (\gamma - \alpha)^2$.

- (b) Solve the equation by Cardon's method

$$x^3 + 6x^2 + 9x + 4 = 0$$

- (c) If $A, B, \dots, L; a, b, \dots, l; m \in R$, then prove that

$$\frac{A^2}{x-a} + \frac{B^2}{x-b} + \dots + \frac{L^2}{x-l} = x + m$$

has all its roots real.

5. Answer either (a) or (b) : 10

- (a) Prove that a mapping $f : X \rightarrow Y$ is one-one onto iff there exists a mapping $g : Y \rightarrow X$ such that $g \circ f$ and $f \circ g$ are identity maps on X and Y , respectively.

- (b) Show that an equivalence relation R in a non-empty set S determines a partition of S and conversely, a partition of S defines an equivalence relation in S .

6. Answer either (a) or (b) :

- (a) If H and K be two subgroups of a group G , then prove that HK is a subgroup of G iff $HK = KH$.
 $[HK = \{hk : h \in H, k \in K\}]$ 10

- (b) Prove that order of each subgroup of a finite group is a divisor of the order of the group. Hence prove that if G is a finite group of order n and $a \in G$, then $a^n = e$. $6+4=10$

(6)

7. Answer either (a) or (b) :

(a) If $\tan(\alpha + i\beta) = x + iy$, then find x and y . Hence show that $x^2 + y^2 + 2x \cot 2\alpha = 1$ 10

(b) (i) If $x < \sqrt{2} - 1$, then prove that

$$2\left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \dots\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^5 - \dots$$

(ii) Show that

$$\frac{\pi}{12} = \left(1 - \frac{1}{3^{1/2}}\right) - \frac{1}{3}\left(1 - \frac{1}{3^{3/2}}\right) + \frac{1}{5}\left(1 - \frac{1}{3^{5/2}}\right) - \dots \infty$$

5+5=10

8. Answer either (a) or (b) :

(a) If A and B are two square matrices of the same order, then prove that

$$\text{adj}(AB) = (\text{adj } B) \cdot (\text{adj } A)$$

Verify it for the matrices

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 6 & 1 \\ -1 & 3 \end{bmatrix} \quad 6+4=10$$

(7)

(b) What is normal form of matrix of a rank r ? Find the rank of the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

by reducing it to normal form. 2+8=10

2022

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 1×10=10

(a) Write n th derivative of $\log(ax + b)$.

(b) If $z = f(y/x)$, what is the value of

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$$

(c) Write the formula for radius of curvature of a Cartesian curve.

(2)

(d) Two curves $y = f(x)$ and $y' = g(x)$ intersect at the point (x_1, y_1) . Find the condition that they cut orthogonally.

(e) Define double point of a curve.

(f) What is the volume of the solid generated due to the revolution of the circle $x^2 + y^2 = 4$ about X -axis?

(g) Write subnormal to the curve $y^2 = 4ax$ at any point (x, y) .

(h) Write the value of $\int_0^{\pi/2} \cos^7 x \, dx$.

(i) Write the value of

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} \, dx$$

(j) Write the maximum number of asymptotes of algebraic curve of n th degree.

(3)

2. Solve the following questions : $2 \times 5 = 10$

(a) If $y = e^{ax} \sin bx$, show that

$$y_2 - 2ay_1 + (a^2 + b^2)y = 0$$

(b) If $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0, \quad x \neq 0, \quad y \neq 0$$

(c) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(d) Find the value of $\int_0^{\pi} x \cos^4 x \, dx$.

(e) Find the equation of tangent to the curve $y = be^{-x/a}$ at the point, where it crosses the axis of y .

3. Answer the following questions : $5 \times 2 = 10$

(a) If $y = \sin^{-1} x$, find $(y_n)_0$ where n is odd.

(b) Obtain a reduction formula for $\int \sec^n x dx$.

4. Answer either part (a) or part (b) : 10

(a) (i) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

(ii) Find the points of inflexion of the curve $y(a^2 + x^2) = x^3$. 5+5=10

(b) (i) If $u = f(x, y)$, where $x = r \cos \theta$, $y = r \sin \theta$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta}\right)^2$$

(ii) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with x -axis, show that its equation is

$$y \cos \phi - x \sin \phi = a \cos 2\phi.$$

5+5=10

5. Answer the following questions : 5×2=10

(a) Evaluate :

$$\int_0^{\pi/2} \log \sin x dx$$

(b) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6. Answer part (a) or part (b) : 10

(a) (i) Obtaining n th derivative of x^{2n} , prove that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \cdot 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \cdot 2^2 \cdot 3^2} + \dots = \frac{12n}{(12n)^2}$$

(ii) Show that the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is

$$\frac{3}{8} \pi a^2.$$

5+5=10

(b) (i) Find the asymptotes of the curve

$$x^3 + y^3 - 3axy = 0$$

(ii) Trace the curve $r = a(1 + \cos \theta)$.

5+5=10

(6)

7. Answer any two questions : $5 \times 2 = 10$

(a) Evaluate :

$$\int_0^{\pi/2} \frac{dx}{5+3\cos x}$$

(b) If $I_n = \int (a^2 + x^2)^{n/2} dx$, show that

$$I_n = \frac{x(a^2 + x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$

(c) Evaluate :

$$\int \frac{dx}{(x^2 - 2x + 1)\sqrt{x^2 - 2x + 3}}$$

8. Answer the following questions : $5 \times 2 = 10$

(a) Integrate :

$$\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$$

Or

$$\int \frac{dx}{(1-x)\sqrt{1-x^2}}$$

(7)

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

Or

Find the surface area of the solid generated by revolving the cardioid $r = a(1 + \cos \theta)$ about the initial line.
