Total number of printed pages–32

3 (Sem–5/CBCS) MAT HE 1/HE 2/HE 3

2022

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5016

(Number Theory)

DSE (H)–1

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

PART–A

1. Choose the correct option in each of the following questions : ***(any ten)*** $1\times10=10$

   (i) Number of integers which are less than and co-prime to 108 is

   (a) 18

   (b) 17

Contd.

(c) 15

(d) 36

(ii) The number of positive divisors of a perfect square number is

(a) odd

(b) even

(c) prime

(d) Can't say

(iii) If $100! \equiv x(mod 101)$, then x is

(a) 99

(b) 100

(c) 101

(d) None of the above

(iv) The solution of pair of linear congruences $2x \equiv 1(mod\ 5)$ and $x \equiv 4\ (mod\ 3)$ is

(a) $x \equiv 13(mod 5)$

(b) $x \equiv 28(mod 5)$

(c) $x \equiv 13(mod 15)$

(d) $x \equiv 13(mod 3)$

(v) If $a = qb$ for some integer q and $a, b \neq 0$, then

(a) b divides a

(b) a divides b

(c) $a = b$

(d) None of the above

(vi) If a and b are any two integers, then there exists some integres x and y such that

(a) $gcd(a,b) = ax + by$

(b) $gcd(a,b) = ax - by$

(c) $gcd(a,b) = ax^n + by^m$

(d) $gcd(a,b) = (ax + by)^n$

(vii) The linear diophantine equation $ax + by = c$ with $d = gcd(a,b)$ has a solution in integers if and only if

(a) $d|c$

(b) $c|d$

(c) $d|(ax+by)$

(d) Both (a) and (c)

(viii) If a positive integer n divides the difference of two integers a and b, then

(a) $a \equiv b(\mathrm{mod}\, n)$

(b) $a = b(\mathrm{mod}\, n)$

(c) $a \equiv n(\mathrm{mod}\, b)$

(d) None of the above

(ix) The set of integers such that every integer is congruent modulo m to exactly one integer of the set is called _____ modulo m. *(Fill in the blank)*

(a) Reduced residue system

(b) Complete residue system

(c) Elementary residue system

(d) None of the above

(x) Which of the following statement is false?

(a) There is no pattern in prime numbers

(b) No formulae for finding prime numbers

(c) Both (a) and (b)

(d) None of the above

(xi) The reduced residue system is ____ of complete residue system.

(a) compliment

(b) subset

(c) not a subset

(d) Both (a) and (c)

(xii) The unit place digit of 137^{93} is

(a) 7

(b) 9

(c) 3

(d) 1

(xiii) Euler phi-function of a prime number p is

(a) p

(b) $p-1$

(c) $p/2-1$

(d) None of the above

(xiv) Which theorem states that "if p is prime, then $(p-1)! \equiv -1 (mod\, p)$" ?

(a) Dirichlet's theorem

(b) Wilson's theorem

(c) Euler's theorem

(d) Fermat's little theorem

(xv) Let p be an odd prime. Then $x^2 \equiv -1 (mod\, p)$ has a solution if p is of the form

(a) $4k+1$

(b) $4k$

(c) $4k+3$

(d) None of the above

(xvi) Let m be a positive integer. Two integers a and b are congruent modulo m if and only if

(a) $m \mid (a-b)$

(b) $m \mid (a+b)$

(c) $m \mid (ab)$

(d) Both (b) and (c)

(xvii) If $ac \equiv bc (mod\, m)$ and $d = gcd(m,c)$

(a) $a \equiv b\left(mod \frac{m}{d}\right)$

(b) $a \equiv c\left(mod \frac{m}{d}\right)$

(c) $a \equiv m (mod\, b)$

(d) $a \equiv m\left(mod \frac{b}{a}\right)$

(xviii) If a is a whole number and p is a prime number, then according to Fermat's theorem

(a) $a^p - a$ is divisible by p

(b) $a^p - 1$ is divisible by p

(c) $a^{p-1} - 1$ is divisible by p

(d) $a^{p-1} - a$ is divisible by p

2. Answer ***any five*** questions : 2×5=10

(a) Find last two digits of 3^{100} in its decimal expansion.

Contd.

(b) If p and q are positive integers such that $gcd(p,q)=1$, then show that $gcd(a+b, a-b)=1$ or 2.

(c) Find the solution of the following linear Diophantine equation $8x-10y=42$.

(d) If p and q are any two real numbers, then prove that $[p]+[q]\leq[p+q]$ (where $[x]$ denotes the greatest integer less *or* equal to x).

(e) If m and n are integers such that $(m,n)=1$, then $\varphi(mn)=\varphi(m)\varphi(n)$.

(f) Find (7056).

(g) If $a\equiv b(mod\, n)$ and $m\mid n$, then show that $a\equiv b(mod\, m)$.

(h) List all primitive roots modulo 7.

(i) If $n=p_1^{k_1}p_2^{k_2}....p_m^{k_m}$ is the prime factorization of $n>1$, then prove that $\tau(n)=(k_1+1)(k_2+1)........(k_n+1)$.

(j) Evaluate the exponent of 7 in 1000 !

3. Answer ***any four*** questions : 5×4=20

(a) If p is a prime, then prove that
$$\varphi(p!)=(p-1)\varphi((p-1)!)$$

(b) Show that, the set of integers $\{1,5,7,11\}$ is a reduced residue system (RRS) modulo 12.

(c) Solve the following simultaneous congruence :
$$x\equiv 2(mod 3)$$
$$x\equiv 2(mod 2)$$
$$x\equiv 3(mod 5)$$

(d) For $n=p^k, p$ is a prime, prove that $n=\sum_{d|n}\varphi(d)$, where $\sum_{d|n}$ denotes the sum over all positive divisors of n.

(e) If p_n is the n^{th} prime, then show that $\frac{1}{p_1}+\frac{1}{p_2}+...+\frac{1}{p_n}$ is not an integer.

(f) Let n be any integer >2. Then $\varphi(n)$ is even.

(g) Show that if $a_1, a_2, ..., a_{\varphi(m)}$ is a RRS modulo m, where m is a positive integer with $m\neq 2$, then $a_1+a_2+...+a_{\varphi(m)}\equiv 0(mod\, m)$.

Contd.

(h) Show that $10!+1$ is divisible by 11.

PART–B

Answer ***any four*** of the following questions : $10\times4=40$

4. (a) If $a, b \neq 0$ and c be any three integers and $d = gcd(a,b)$. Then show that $ax+by=c$ has a solution iff $d\,|\,c$.

Furthermore, show that if x_0 and y_0 is a particular solution of $ax+by=c$, then any other solution of the equation is $x' = x_0 - \frac{b}{d}t$ and $y' = y_0 + \frac{a}{d}t$, t is an integer. 7

(b) Find the general solution of $10x - 8y = 42;\ x, y \in Z$ 3

5. (a) Show that an odd prime p can be represented as sum of two squares iff $p \equiv 1(mod 4)$. 7

(b) Find all positive solutions of $x^2 + y^2 = z^2$, where $0 < z < 30$. 3

6. (a) Prove that the quadratic congruence $x^2+1 \equiv 0(mod\,p)$, where p is an odd prime, has a solution iff $p \equiv 1(mod 4)$. 5

(b) If p is prime and a is an integer not divisible by p, prove that $a^{p-1} \equiv 1\,(mod\,p)$. 5

7. State and prove Chinese remainder theorem. Also find all integers that leave a remainder of 4 when divided by 11 and leaves a remainder of 3 when divided by 17.

8. (a) For each positive integer $n \geq 1$, show that $\sum_{d|n} \mu(d) = \begin{cases} 1, \text{if } n=1 \\ 0, \text{if } n>1 \end{cases}$ 5

(b) If k denotes the number of distinct prime factors of positive integer n. Prove that $\sum_{d|n} |\mu(d)| = 2^k$ 5

9. (a) If p is a prime, prove that $\varphi(p^k) = p^k - p^{k-1}$, for any positive integer k. For $n > 2$, show that $\varphi(n)$ is an even integer. 3+2=5

(b) State Mobius inversion formula. If the integer $n>1$ has the prime factorization. If $n=p_1^{k_1}p_2^{k_2}....p_s^{k_s}$, then prove that

$$\sum_{d|n}\mu(d)\,\sigma(d)=(-1)^s p_1\,p_2\cdots p_s.$$ 5

10. If x be any real number. Then show that 1+3+3+3=10

(a) $[x]\le x<[x]+1$

(b) $[x+m]=[x]+m$, m is any integer

(c) $[x]+[-x]=\begin{cases}0, & \text{if } x \text{ is an integer}\\ -1, & \text{otherwise}\end{cases}$

(d) $\left[\frac{[x]}{m}\right]=\left[\frac{x}{m}\right]$, if m is a positive integer

11. (a) If $a_1, a_2,..., a_m$ is a complete residue system modulo m, and if k is a positive integer with $(k,m)=1$ then $ka_1+b,\ ka_2+b,\ ...,\ ka_m+b$, is a complete residue system modulo m for any integer b. 5

(b) Examine whether the following set forms a complete residue system or a reduced residue system :

$\{-3,14,3,12,37,56,-1\}(mod\,7)$ 5

12. (a) If $n\ge1$ is an integer then show that

$$\prod_{d|n} d=n^{\frac{\tau(n)}{2}}$$ 3

(b) If f and g are two arithmetic functions, then show that the following conditions are equivalent : 7

(i) $f(n)=\sum_{d|n} g(d)$

(ii) $g(n)=\sum_{d|n}\mu(d)f\left(\frac{n}{d}\right)=\sum_{d|n}\mu\left(\frac{n}{d}\right)f(d)$

13. (a) If n is a positive integer with $n\ge2$, such that $(n-1)!+1\equiv0\ (mod\ n)$, then show that n is prime. 5

(b) Show that if p is an odd prime, then $2(p-3)!\equiv-1(mod\,p)$. 5

OPTION–B

Paper : MAT–HE–5026

(Mechanics)

1. Answer the following questions : ***(any ten)*** 1×10=10

(i) If a system of coplanar forces is in equilibrium, then what is the algebraic sum of the moment of the forces about any point in the plane ?

(ii) What is the resultant of the like parallel forces $P_1, P_2, P_3, ...$ acting on a body ?

(iii) If a particle moves under the action of a conservative system of forces, then what is the sum of its KE and PE ?

(iv) Define limiting equilibrium.

(v) Define the centre of gravity of a body.

(vi) Under what conditions the effect of a couple is not altered if it is transformed to a parallel plane ?

(vii) Write down the radial and cross-radial components of velocities of a particle moving on a plane curve at any point (r, θ) on it.

(viii) What is the resultant of a couple and a force in the same plane ?

(ix) What is dynamical friction ?

(x) What do you mean by terminal velocity ?

(xi) Define coefficient of friction.

(xii) What is the position of the point of action of the resultant of two equal like parallel forces acting on a rigid body ?

(xiii) What is the whole effect of a couple acting on a body ?

(xiv) Define simple harmonic motion.

(xv) What is the centre of gravity of a triangular lamina ?

(xvi) Define limiting friction.

(xvii) State the principle of conservation of energy.

(xviii) A particle moves on a straight line towards a fixed point O with an acceleration proportional to its distance from O. If x is the distance of the particle at time t from O, then write down its equation of motion.

2. Answer ***any five*** questions of the followoing : 2×5=10

(a) Write the laws of static friction.

(b) A particle moves in a circle of radius r with a speed v. Prove that its angular velocity is $\frac{v}{r}$.

(c) What are the general conditions of equilibrium of any system of coplanar forces ?

(d) The law of motion in a straight line is $s=\frac{1}{2}vt$. Prove that the acceleration is constant.

(e) Find the greatest and least resultant of two forces acting at a point whose magnitudes are P and Q respectively.

(f) Find the centre of gravity of an arc of a plane curve $y=f(x)$.

(g) State Hooke's law.

(h) Show that impulse of a force is equal to the momentum generated by the force in the given time.

(i) Write the expression for the component of velocity and acceleration along radial and cross radial direction in a motion of a particle in a plane curve.

(j) The speed v of a particle moving along x-axis is given by the relation $v^2=n^2\left(8bx-x^2-12b^2\right)$. Prove that the motion is Simple Harmonic.

3. Answer ***any four*** questions of the following : 5×4=20

(a) The greatest and least resultants that two forces acting at a point can have magnitude P and Q respectively. Show that when they act at an angle α their resultant is $\sqrt{P^2\cos^2\frac{\alpha}{2}+Q^2\sin^2\frac{\alpha}{2}}$.

(b) I is the in centre of the triangle ABC. If three forces $\vec{P},\vec{Q},\vec{R}$ acting at I along $\overrightarrow{IA}, \overrightarrow{IB}, \overrightarrow{IC}$ are in equilibrium, prove that

$$\frac{P}{\sqrt{a(b+c-a)}}=\frac{Q}{\sqrt{b(c+a-b)}}=\frac{R}{\sqrt{c(a+b-c)}}$$

(c) Show that the resultant of three equal like parallel forces acting at the three vertices of a triangle passes through the centroid of the triangle.

(d) Prove that any system of coplanar forces acting on a rigid body can ultimately be reduced to a single force acting at any arbitrarily chosen point in the plane, together with a couple.

(e) Show that the sum of the Kinetic energy and Potential energy is constant throughout the motion when a particle of mass m falls from rest at a height h above ground.

(f) A point moves along a circle with constant speed. Find its angular velocity and acceleration about any point of the circle.

(g) Show that the work done against tension in stretching a light elastic string is equal to the product of its extension and the mean of the initial and final tension.

(h) A particle starts with velocity u and moves under retardation μ times of the distance. Show that the distance it travels before it comes to rest is $\frac{u}{\sqrt{\mu}}$.

4. Answer ***any four*** questions of the following : 10×4=40

(a) Forces P, Q and R act along the sides BC, CA and AB of a triangle ABC and forces P', Q' and R' act along OA, OB and OC, where O is the centre of the circumscribed circle, prove that

(i) $P\cos A + Q\cos B + R\cos C = 0$

(ii) $\frac{PP'}{a} + \frac{QQ'}{b} + \frac{RR'}{c} = 0$

(b) State and prove Lami's theorem. Forces P, Q and R acting along OA, OB and OC, where O is the circumcentre of triangle ABC, are in equilibrium. Show that

$$\frac{P}{a^2\left(b^2+c^2-a^2\right)} = \frac{Q}{b^2\left(c^2+a^2-b^2\right)} = \frac{R}{c^2\left(a^2+b^2-c^2\right)}$$

(c) (i) Find the centre of gravity of a uniform arc of the circle $x^2+y^2=a^2$ in the positive quadrant.

Contd.

(ii) Find the centre of gravity of the arc of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant.

(d) A particle moves in a straight line under an attraction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Investigate the motion.

(e) A particle moves in a straight line OA starting from the rest at A and moving with an acceleration which is directed towards O and varies as the distance from O. Discuss the motion of the particle. Hence define Simple Harmonic Motion and time period of the motion.

(f) Find the component of acceleration of a point moving in a plane curve along the initial line and the radius vector. Also find the component of acceleration perpendicular to initial line and perpendicular to radius vector.

(g) A particle is falling under gravity in a medium whose resistance varies as the velocity. Find the distance and velocity at any time t. Also find the terminal velocity of the particle.

(h) The velocity component of a particle along and perpendicular to the radius vector from λr and $\mu\theta$. Find the path and show that radial and transverse component of acceleration are

$$\lambda^2 r - \frac{\mu^2\theta^2}{r} \text{ and } \mu\theta\left(\lambda + \frac{\mu}{r}\right).$$

(i) Find the component of acceleration of a point moving in a plane curve along the initial line and the radius vector. Also find the component of acceleration perpendicular to initial line and perpendicular to radius vector.

(j) A particle moves in a straight line under an attaction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Investigate the motion.

OPTION–C

Paper : MAT–HE–5036

(Probability and Statistics)

1. Answer ***any ten*** questions from the following : 1×10=10

(a) If A and B are mutually exclusive what will be the modified statement of

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(b) Define probability density funciton for a continuous random variable.

(c) A random variable X can take all non-negative integral values, and the probability that X takes the value r is $P(X = r) = A\alpha^r (0 < \alpha < 1)$. Find $P(X = 0)$.

(d) If X and Y are two random variables and $var\ (X - Y) \neq var(X) - var(Y)$ then what is the relation between X and Y.

(e) Test the velocity of the following probability distribution :

x	-1	0	1
$P(x)$	0.4	0.4	0.3

(f) Define Negative Binomial distribution for a random variable X with parameter r.

(g) What are the relations between mean, median and mode of a normal distribution ?

(h) Write the equation of the line of regression of x on y.

(i) What is the variance of the mean of a random sample ?

(j) Define moment generating function of a random variable X about origin.

(k) What are the limits for correlation coefficents ?

(l) For a Bernoulli random variable X with $P(X = 0) = 1 - P$ and $P(X = 1) = P$ write $E(X)$ and $V(X)$ in terms of P.

(m) If X is a random variable with mean μ and variance σ^2, then for any positive number k, find Chebychev's inequality.

(n) A continuous random variable X follow the probability law $f(x) = Ax^2, 0 \leq x \leq 1$. Determine A.

(o) If X and Y are two random variables then find $cov(x, y)$.

(p) If a is constant then find $E(a)$ and $var(a)$.

(q) If X and Y are two independent. Poisson variates, then XY is a ______ variate.
(Fill in the blank)

(r) If a non-negative real valued function f is the probability density function of some continuous random variable, then what is the value of $\int_{-\alpha}^{\alpha} f(x)\,dx$?

2. Answer the following questions : ***(any five)*** 2×5=10

(a) If A and B are independent events, then show that A and B are also independent.

(b) If X have the p.m.f

$$f(x) = \frac{x}{10}, x = 1,2,3,4$$

then find $E(X^2)$

(c) With usual notation for a binomial variate X, given that $9\,P(x=4) = P(x=2)$ when $n=6$. Find the value of p and q.

(d) If X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} \frac{3}{8}(4x - 2x^2) & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

then find $P\{X > 1\}$.

(e) Show that for a normal standard variate z, $E(z) = 0$ and $V(z) = 1$.

(f) The number of items produced in a factory during a week is a random variable with mean 50. If the variance of a week's production is 25, then what is the probability that this week's production will be between 40 and 60.

(g) Define probability mass function and probability density function for a random varibale X.

Contd.

(h) If X is a random Poisson variate with parameter λ, then show that

$$P(X \geq n) - P(X \geq n+1) = \frac{e^{-\lambda} m^n}{\lfloor n}$$

(i) If $M_x(t)$ is a moment generating function of a random variable X with parameter t then show that $M_{cX}(t) = M_X(ct)$, c is a constant.

(j) If X and Y are independent random variables with characteristic functions $\varphi_X(w)$ and $\varphi_Y(w)$ respectively then show that

$$\varphi_{X+Y}(w) = \varphi_X(w)\varphi_Y(w)$$

3. Answer ***any four*** questions from the following : 5×4=20

(a) If X is a discrete random variable having probability mass function 2+2+1=5

mass point	0	1	2	3	4	5	6	7
$p(X=x)$	0	k	$2k$	$3k$	$4k$	k^2	$2k^2$	$7k^2+k$

Determine :

(i) k

(ii) $p(X<6)$ and

(iii) $p(X \geq 6)$

(b) If X and Y are two independent random variables then show that

$$var(X+Y) = var(X) + var(Y)$$

(c) If the probability that an individual will suffer a bad reaction from injective of a given serum is 0.001 determine the probability that out of 2000 individuals

(i) exactly 3,

(ii) more than 2 individual

will suffer a bad reaction. 2+3=5

(d) Two random variables X and Y are jointly distributed as follows :

$$f(x,y) = \frac{2}{\pi}\left(1 - x^2 - y^2\right),\ 0 < x^2 + y^2 < 1$$

Find the marginal distribution of X.

(e) State and prove weak law of large numbers.

(f) If X and Y are independent random variables having common density function

$$f(x) = e^{-x}, x > 0$$
0, otherwise

Find the density function of the random variable X/Y.

(g) If X and Y are independent Poisson variates such that

$P(x=1)=P(x=2)$ and

$P(y=2)=P(y=3)$

Find the variance of $x-2y$.

(h) Prove that regression coefficients are independent of the change of origin but not of scale.

4. Answer ***any four*** from the following questions : 10×4=40

(a) (i) What is meant by partition of a sample space S? If $Hi\,(i=1,2,..n)$ is a partition of the sample space S, then for any event A, prove that

$$P(Hi/A)=\frac{P(Hi)P(A/Hi)}{\sum_{i=1}^{n}P(Hi)P(A/Hi)}$$ 5

(ii) If X is a random variable with the following probability distribution :

x:	-3	6	9
$P(X=x)$:	$1/6$	$1/2$	$1/3$

Find $E(X), E(X^2)$ and $var(X)$ 5

(b) Two random variables X and Y have the following joint probability density function : 2+2+3+3=10

$$f(x,y)=\begin{cases}2-x-y, & 0\le x\le 1,\ 0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$$

Find (i) marginal probability density function of X and Y (ii) conditional density function (iii) var (X) and var (Y) (iv) co-variance between X and Y.

(c) (i) Let X be a random variable with mean μ and variance r^2. Show that $E(x-b)^2$ as a function of b is minimum when $b=\mu$. 5

(ii) A bag contains 5 balls and it is known how many of these are white. Two balls are drawn and are found to be white. What is the probability that all are white ? 5

(d) (i) If X is a random variable then prove that 5

$$var(X)=var[E(X/Y)]+E[var(X/Y)]$$

(ii) Find the probability that in a family of 4 children there will be *(a) at least one* boy *(b) at least one* boy and *at least one* girl. 5

(e) What are the chief characteristics of the normal distribution and normal curve ?

(f) *(i)* Show that mean and variance of a Poisson distribution are equal. 5

(ii) Determine the binomial distribution for which the mean is 4 and variance is 3 and find its mode. 5

(g) *(i)* Prove that independent variables are uncorrelated. With the help of an example show that the converse is not true. 5

(ii) Find the angle between the two lines of regression 5

$$y - \overline{y} = \frac{r\sigma_y}{\sigma_x}(x - \overline{x})$$

and $x - \overline{x} = r\frac{\sigma_x}{\sigma_y}(y - \overline{y})$

(h) *(i)* A function $f(x)$ of x is defined as follows :

$$f(x) = 0 \text{ for } x < 2$$
$$= \frac{1}{18}(3 + 2x) \text{ for } 2 \le x \le 4$$
$$= 0, \text{ for } x > 4$$

Show that it is a density function. Also find the probability that a variate with this density will lie in the interval $2 \le x \le 3$. 5

(ii) A random variable X can assume values 1 and -1 with probability $\frac{1}{2}$ each. Find

(i) moment generating function,

(ii) characteristics function. 5

(i) *(i)* Find the median of a normal distribution. 5

(ii) A random variable X has density function given by 5

$$f(x) = \begin{cases} 2e^{-2x}, x \ge 0 \\ 0, \quad x < 0 \end{cases}$$

Find *(i)* mean with the help of m.g.f. *(ii)* $P[|x - \mu| > 1]$.

Contd.

(j) *(i)* The diameter say x, of an electric cable is assumed to be continuous random variable with p.d.f

$$f(x) = 6x(1-x),\ 0 \le x \le 1$$

(a) Check that the above is a p.d.f,

(b) Determine the value of k such that $P(X < K) = P(X > K)$ 5

(ii) If 3% of electric bulbs manufactured by a company are defective, using Poisson's distribution find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective 5

$$\left[\text{Given } e^{-3} = 0.04979\right]$$

Total number of printed pages–31

3 (Sem–5/CBCS) MAT HE 4/5/6

2022

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE(H)–2

Full Marks : 80

Time : Three hours

OPTION–B

Paper : MAT–HE–5056

(Spherical Trigonometry and Astronomy)

Full Marks : 80

Time : Three hours

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

Contd.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE (H)–2

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** questions from the following : *(Choose the correct answer)* $1\times10=10$

(i) A function $f(x)$ is said to the strictly convex at x if for two other distinct points x_1 and x_2

(a) $f[\lambda x_1+(1-\lambda)x_2]<\lambda f(x_1)+(1-\lambda)f(x_2)$, where $0\leq\lambda\leq1$

(b) $f[\lambda x_1+(1-\lambda)x_2]<\lambda f(x_1)+(1-\lambda)f(x_2)$, where $0<\lambda<1$

(c) $f[\lambda x_1+(1-\lambda)x_2]\leq\lambda f(x_1)+(1-\lambda)f(x_2)$, where $0\leq\lambda\leq1$

(d) None of the above

(ii) If X is the set of eight vertices of a cube, then the convex hull $C(X)$ is the

(a) surface of the cube

(b) vertices of the cubic

(c) whole cube

(d) None of the above

(iii) The extreme points of the convex set of feasible solutions are

(a) finite in number

(b) infinite in number

(c) either finite *or* infinite

(d) None of the above

(iv) In a linear programming problem

$$max\ Z=cx$$

subject to $Ax\geq b,\ x\geq0$, c is called

(a) coefficient vector

(b) column vector

(c) price vector

(d) None of the above

(v) Consider a system $Ax = b$ of m equations in n unknowns, $n > m$. Then maximum number of basic solution is

(a) ${}^{m}C_{n}$

(b) ${}^{n}C_{m-1}$

(c) ${}^{n}C_{n-m}$

(d) None of the above

(vi) A basic feasible solution of an LPP is said to be non-degenerate BFS if

(a) none of the basic variable zero

(b) at least one of the basic variable zero

(c) exactly one of the basic variable zero

(d) None of the above

(vii) If the LPP $max\ Z = cx$ such that $Ax = b, x \geq 0$ has a feasible solution then at least one of the BFS will be

(a) maximal

(b) minimal

(c) optimal

(d) None of the above

(viii) If for any basic feasible solution of an LPP, there is some column α_j in A but not in B for $c_j - z_j > 0$ and $y_{ij} \leq 0\ (i = 1, 2, \cdots, m)$, then the problem has an unbounded solution if the objective function is to be

(a) maximized

(b) minimized

(c) *either* maximized *or* minimized

(d) None of the above

(ix) Standared form of LPP is

(a) Min $Z = cx$ s.t. $Ax \geq b,\ x \geq 0$

(b) Max $Z = cx$ s.t. $Ax \leq b,\ x \geq 0$

(c) Max $Z = cx$ s.t. $Ax \geq b,\ x \geq 0$

(d) None of the above

(x) The incoming vector in a simplex table will be taken as α_k if

(a) $\Delta_k = max\ \Delta_j$

(b) $\Delta_j = max\ \Delta_k$

(c) entries of α_k are all negative

(d) None of the above

(xi) If we consider dual of an LPP, then in the dual the requirement vector of the primal problem becomes

(a) objective function

(b) price vector

(c) variable

(d) None of the above

(xii) The necessary and sufficient condition for any LPP and its dual to have optimal solution is that

(a) both have basic solution

(b) both have unbounded solution

(c) both have feasible solution

(d) None of the above

(xiii) If any of the constraint in the primal is perfect equality, the corresponding dual variable is

(a) perfect equality

(b) unrestricted in sign

(c) strictly inequality

(d) None of the above

(xiv) In an assignment problem

$$min\,Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij},$$

$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \cdots, n$ means that

(a) only one job is done by i-th person $i = 1, 2, \cdots, n$

(b) i-th person is assigned to j-th job

(c) only one person should be assigned to the j-th job, $j = 1, 2, \cdots, n$

(d) None of the above

(xv) In a transportation problem if we apply North-West Corner method, we always get

(a) non-degenerated BFS

(b) degenerated BFS

(c) optimal solution

(d) None of the above

(xvi) For optimality test in a transportation problem, number of allocation in independent position must be

(a) $m+n$

(b) $m+n+1$

(c) $m+n-1$

(d) None of the above

(xvii) In a transportation table for cell evaluation we use the formula

(a) $c_{rs} = u_r + v_s,\ d_{ij} = (u_i + v_j) - c_{ij}$

(b) $c_{rs} = u_r + v_s,\ d_{ij} = c_{ij} - (u_i + v_j)$

(c) $c_{rs} = u_r + v_s,\ d_{ij} = u_i + v_j$

(d) None of the above

(xviii) Define finite game a "Game Theory".

2. Answer **any five** from the following: 2×5=10

(a) Show that the FS $x_1 = 1,\ x_2 = 0,\ x_3 = 1$ and $z = 6$ to the system of equations

$$x_1 + x_2 + x_3 = 2$$

$$x_1 - x_2 + x_3 = 2,\ x_j \geq 0$$ which minimize

$$z = 2x_1 + 3x_2 + 4x_3$$ is not basic.

(b) Is $x_1 = 1,\ x_2 = \frac{1}{2},\ x_3 = x_4 = x_5 = 0$ a basic solution to the following system?

$$x_1 + 2x_2 + x_3 + x_4 = 2$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 + x_5 = 2$$

(c) Examine convexity of the set

$$S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\}$$

(d) Define artificial variable. Give an example.

(e) Define unbounded solution of an LPP. How can we determine that the solution of an LPP is unbounded?

(f) What is two phase method to solve an LPP? Mention the phases?

(g) Write "complementary slackness theorem" of a dual problem.

(h) How can we find entering vector in a simplex table?

(i) Write the 'Test of optimality' for primal dual method.

(j) What is cost matrix of an assignment problem?

3. Answer ***any four*** from the following : 5×4=20

(a) Prove that the set of all feasible solutions of an LPP is a convex set. [Assume that the set is non empty]

(b) If the objective function of an LPP assume its optimal value at more than one extreme point, then prove that every convex combination of these extreme points gives the optimal value of the objective function.

(c) Give the dual of the following LPP :

$$Max\ Z = 2x_1 + 3x_2 + x_3$$

$$\text{s.t.}\quad 4x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

(d) Solve the following transportation problem by North-West Corner method

	S_1	S_2	S_3	S_4	a_i
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
b_j	20	40	30	40	100

(e) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

(f) Mark the feasible region represented by the constraint conditions

$$x_1 + x_2 \leq 1,\ 3x_1 + x_2 \geq 3,\ x_1 \geq 0,\ x_2 \geq 0$$

(g) Find initial BFS of the following LPP :

$$Max\ Z = 2x_1 + 3x_2$$

$$\text{s.t.}\quad -x_1 + 2x_2 \leq 4$$

$$x_1 + x_2 \leq 6$$

$$x_1 + 3x_2 \leq 9$$

$$x_1, x_2 \text{ unrestricted}$$

(h) If in an assignment problem, a constant is added or substracted to every element of row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimize the total cost for the other matrix.

Contd.

4. Answer ***any four*** from the following : 10×4=40

(a) A soft drink plant has two bottling machines A and B. It produces and sells 8 ounce and 16 ounce bottles. The following data is available

Machine	8 ounce	16 ounce
A	100/minute	40/minute
B	60/minute	75/minute

The machines can run 8 hours per day, 5 days per week. Weekly production of drink cannot exceed 3,00,000 ounces and the market can absorb 25,000 eight ounce bottles and 7,000 sixteen ounce bottles per week, Profit on these bottle is 15 paise and 25 paise per bottle respectively. The planner wishes to minimize his profit subject to all the production and marketing restrictions. Formulate it as an LPP and solve graphically.

(b) State and prove fundamental theorem of LPP.

(c) Using simplex alogorithm solve the problem

$$Max\ Z = 2x_1 + 5x_2 + 7x_3$$

$$\text{s.t. } 3x_1 + 2x_2 + 4x_3 \le 100$$
$$x_1 + 4x_2 + 2x_2 \le 100$$
$$x_1 + x_2 + 3x_3 \le 100$$
$$x_1, x_2, x_3 \ge 0$$

(d) Use dual to solve the LPP

$$Min\ Z = 2x_1 + x_2$$

$$\text{s.t. } 3x_1 + x_2 \ge 3$$
$$4x_1 + 3x_2 \ge 6$$
$$x_1 + 2x_2 \ge 3$$
$$x_1, x_2 \ge 0$$

(e) Solve the following transportation problem :

Market

Plant	A	B	C	D	Available
X	10	22	10	20	8
Y	15	20	12	8	13
Z	20	12	10	15	11
Required	5	11	8	8	32

(f) State and prove Fundamental Duality theorem.

(g) The pay-off matrix for A in a two persons zero sum game is given below. Determine the value of the game and the optimum strategies for both players

		B: I	II	III
A	I	−1	2	1
	II	1	−2	2
	III	3	4	−3

(h) Find an optimal solution of the following LPP without using simplex method :

$$Max\ Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

$$\text{s.t.}\quad 2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_i \geq 0,\ i = 1, 2, 3, 4$$

(i) If an LPP

$$Max\ Z = cx \quad \text{s.t.}\ Ax = b,\ x \geq 0$$

where A is $mX(m+n)$ matrix of co-efficients given by $A = (\alpha_1, \alpha_2, \cdots, \alpha_{m+n})$, has at least one feasible solution, then prove that it has at least one basic feasible solution.

(j) A company has four territories open and four salesman available for assignment. The territories are not equally rich in their sales potential; it is estimated that a typical salesman operating in each territory would bring in the following annual sales

Territories	I	II	III	IV
Annual sales (*Rs.*)	60,000	50,000	40,000	30,000

The four salesman are also considered to differ in ability; it is estimated that, working under the same conditions, their yearly sales would be proportionally as follows :

Salesman :	A	B	C	D
Proportion :	7	5	5	4

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory, the next best salesman to the second richest and so on. Verify this answer by the assignment technique.

Contd.

OPTION–B

Paper : MAT–HE–5056

(*Spherical Trigonometry and Astronomy*)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any ten*** of the following questions : 1×10=10

(i) Define spherical triangle.

(ii) Define great circle and small circle.

(iii) How many great circles can be drawn through two given points, when the points are the extremities of a diameter ?

(iv) What are the relations between the elements of a spherical triangle and that of its polar triangle ?

(v) Define hour angle of a heavenly body.

(vi) What is the point on the celestial sphere whose latitude, longitude, right ascension and declination, all are zero ?

(vii) Name the *two* points in which the elliptic cuts the equator on the celestial sphere.

(viii) What is the declination of the pole of the ecliptic ?

(ix) What is parallatic ellipse ?

(x) What do you mean by circumpolar star ?

(xi) State the third law of Kepler.

(xii) Define polar triangle.

(xiii) What is the duration of a day and night at equinoxes ?

(xiv) Explain, what is meant by rising and setting of stars ?

(xv) Where does the celestial equator cut the horizon ?

(xvi) Define right ascension of a heavenly body.

(xvii) What are the altitude and hour angle of the zenith ?

(xviii) State the cosine formula related to a spherical triangle.

2. Answer ***any five*** of the following questions : 2×5=10

(a) Draw a neat diagram of the celestial sphere showing the horizontal coordinates of a heavenly body.

(b) Prove that the sides and angles of a polar triangle are respectively the supplements of the angles and sides of the primitive triangle.

(c) State Newton's law of gravitation.

(d) ABC is an equilateral spherical triangle, show that $\sec A = 1 + \sec a$.

(e) Give the usual three methods for locating the position of a star in space.

(f) Prove that the altitude of the celestial pole at any place is equal to the latitude of the place of the observer.

(g) Discuss the effect of refraction on sunrise.

(h) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

(i) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

(j) Show that right ascension α and declination δ of the sun is always connected by the equation $\tan \delta = \tan \varepsilon \sin \alpha$, ε being obliquity of the ecliptic.

3. Answer ***any four*** questions of the following : 5×4=20

(a) In a spherical triangle ABC, prove that $\cos a = \cos b \cos c + \sin b \sin c \cos A$

(b) In a spherical triangle ABC, if $b + c = \pi$, then prove that $\sin 2B + \sin 2C = 0$.

(c) At a place in north latitude ϕ, two stars A and B of declinations δ and δ_1 respectively, rise at the same moment and A transists when B sets. Prove that $\tan\phi \tan\delta = 1 - 2\tan^2\phi \tan^2\delta_1$

(d) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi \sec\delta$, where the symbols have their usual meanings.

Contd.

(e) Deduce Kepler's laws from the Newton's law of gravitation.

(f) Write short notes on : (i) Zodiac (ii) Morning star and Evening star.

(g) If v_1, v_2 be the velocities of two planets in their orbits, and T_1, T_2, be the respective distances from the sun, prove that $v_2 : v_1 = \sqrt{T_1} : \sqrt{T_2}$.

(h) Prove that the altitude of a star is the greatest when it is on the meridian of the observer.

4. Answer ***any four*** questions of the following : 10×4=40

(a) In a spherical triangle ABC, prove that

$$\frac{\sin a}{\sin A} = \sqrt{\frac{1 - \cos a \cos b \cos c}{1 + \cos A \cos B \cos C}}$$

(b) If ψ is the angle at the centre of the sun subtended by the line joining two planets at distance a and b from the sun at stationary points, show that

$$\cos\psi = \frac{\sqrt{ab}}{a + b - \sqrt{ab}}$$

(c) If the inferior ecliptic limits are $\pm\varepsilon$ and if the satellite revolves n times as fast as the sun, and its node regrades θ every revolution the satellite makes round its primary, prove that there cannot be fewer consecutive solar eclipses at one node then the integer next less than $\frac{2(n-1)\varepsilon}{n\theta + 2\pi}$.

(d) What is Cassin's hypothesis ? Under this hypothesis, show that the amount of refraction R can be found from

$$\tan\phi = \frac{\sin R}{\mu - \cos R}$$

where μ is the refractive index of the atmosphere with respect to vacuum and ϕ is the angle of refraction at certain point on the upper surface of the atmosphere.

(e) Explain the effects of refraction on right ascension and declination.

(f) State Kepler's laws of planetary motion. If V_1 and V_2 are the linear velocities of a planet at perihelion and aphelion respectively and e is the eccentricity of the planets orbit, prove that

$$(1 - e)V_1 = (1 + e)V_2.$$

Contd.

(g) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k \tan \xi$.

ξ being the apparent zenith distance of a heavenly body. Mention *one* limitation of this formula.

(h) A star of declination δ is seen on the prime vertical. Show that its declination is increased by $\frac{2K\left(\sin^2\phi - \sin^2\delta\right)}{\sin 2\delta}$ due to refraction, ϕ being the latitude of the place.

(i) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$

Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

(j) Prove that, if the fourth and higher powers of e are neglected,

$$E = M + \frac{e \sin M}{1 - e\cos M} - \frac{1}{2}\left(\frac{e \sin M}{1 - e\cos M}\right)^3$$

is a solution of Kepler's equation in the form.

———

Contd.

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** questions : 1×7=7

(a) Write the output of a:

```
int a;
a=5/2;
```

(b) Write one arithmetic and one logical operator in C.

(c) What is a global variable?

(d) Name the header file that is used to compile the function '*sqrt* (x)'.

(e) Which of the following can be used as a variable : *x*1, *x*_1, *x*%1 ?

(f) Write *two* reserved words used in C language.

(g) Convert the following mathematical expression into a C expression :

$$z = \frac{5x+6}{3x^2+2} - \frac{\sin x^2}{\sqrt{x}}$$

(h) State whether True **or** False : C-language is case-sensitive.

(i) Write *any two* built-in functions used in C-language.

(j) For $x=5$, $y=2$, write the output of $x \% y$.

(k) Write the utility of getch ( ) function.

(l) Define a *two*-dimensional array.

2. Answer ***any four*** questions : 2×4=8

(a) What is the difference between C character and C string?

(b) Write *four* different C statements each adding 1 to integer variable x.

(c) Name *any four* functions available in 'stdio.h'.

(d) Write a C program that will input a character and give output, the same.

(e) int a, b, temp;

$a = 5;$

$b = 3;$

temp $= a;$

$a = b;$

$b =$ temp;

Write the output of 'a' and 'b'.

(f) Write the general syntax of *scanf*( ) function to read the integer variable a.

(g) Write the syntax of 'nested if' statement in C language.

(h) Write the output of the following:

$c = 0$

for $(i = 1; i \le 5; i++)$

$c = c + i;$

3. Answer ***any three*** parts: 5×3=15

(a) Write a C program to calculate the commission for a sales representative as per the sales amount given below:

if sales ≤ 500, commission is 5% of sales

if sales > 500 but ≤ 2000, commission is Rs. 35 plus 10% above Rs. 500 of sales

if sales > 2000 but ≤ 5000, commission is Rs. 185 plus 12% above Rs. 2000 of sales

if sales > 5000, commission is 12.5% of sales

(b) Write a C program to find the average of best three marks from the given four test marks.

Contd.

(c) Give a general syntax of 'switch' statement in C.

Write the outputs of a and b of the following:

(i)	(ii)	(iii)
$a = 5;$	$a = 5;$	$a = 5;$
$b = 7;$	$b = 7;$	$b = 7;$
$if\ (a > b)$	$if\ (a > b)$	$\text{if } (a > b \,\|\, a < b)$
$\{a = a + 1;$	$a = a + 1;$	$a = a + 1;$
$b = b + 1\};$	$b = b + 1$	

(d) Write a C program to print integers from 1 to n omitting those integers which are divisible by 7.

(e) Write a C program to generate the Fibonacci series up to n terms.

(f) Write a C program to find the sum of squares of all integers between 1 and n.

(g) Write a C program to print the $n \times n$ zero marix.

(h) Write a C program to add 1 to each element of a 3×3 matrix.

4. Answer ***any three*** parts: 10×3=30

(a) Write the differences between 'while loop' and 'do-while' loop using examples. Write a C program to check whether the given number is an Armstrong number. (An Armstrong number is one that is equal to the sum of cubes of individual digits. For *ex.* $153 = 1^3 + 5^3 + 3^3$) 5+5=10

(b) Develop a C program to compute the value of π from the series

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \cdots$$

Write a C program to convert a binary number into a decimal number. 5+5=10

(c) Write a C program for each of the following: 5+5=10

(i) to find the mean and standard deviation of any n values.

(ii) to add two matrices of order $m \times n$.

(d) Write a C program to compute the value of e^x using the series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

For this build two functions—one to find the factorial and the other to compute x^n, for a given n.

(e) Write a C program to find the LCM of two numbers a and b, where b is the sum of the digits of a. Use two functions—one is to find LCM and the other is to find the sum of the digits.

($gcd.lcm = a.b$) 10

(f) Write the syntax of 'nested for' loop and show with a suitable C program. What are the differences between 'break' statement and '*exit*( )' function. Write a C program using 'break' statement, and write the outputs. Also write the outputs of the same program if the 'break' statement is replaced by '*exit*( )' function. 1+4+2+3=10

(g) What is meant by recursive function? What is its use? Demonstrate the use of recursive function by a suitable C program. 2+2+6=10

(h) What are the uses of 'continue' and 'goto' statements in a C program? Explain each with a suitable C program segment. 5+5=10

2700