3 (Sem–3/CBCS) PHY HC 1

2022

PHYSICS

(Honours)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : 1×7=7

   (a) Define the singular point of a second order linear differential equation.

   (b) If $P_n(x)$ and $Q_n(x)$ are two independent solutions of Legendre equation, then write the general solution of the Legendre equation.

   (c) Give one example where Hermite polynomial is used in physics.

(d) The function $P_n(1)$ is given as

  (i) zero

  (ii) -1

  (iii) $P_n(-1)$

  (iv) 1

  (Choose the correct option)

(e) Define trace of a matrix.

(f) What is the rank of a zero matrix ?

(g) Define self-adjoint matrix.

(h) What do you mean by eigenvector ?

(i) Which one of the following represents an equation of a vibrating string ?

  (i) $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$

  (ii) $\dfrac{\partial y}{\partial t} = c\dfrac{\partial y}{\partial x}$

  (iii) None of the above

  (Choose the correct option)

(j) Write the Laplace equation spherical polar co-ordinate system.

(k) Define gamma function.

(l) State the Dirichlet condition for Fourier series.

2. Answer **any four** of the following questions :

  2×4=8

(a) Check whether Frobenius method can be applied or not to the following equation :

$$2x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(b) If $\displaystyle\int_{-1}^{+1} P_n(x)dx = 2$, find the value of n.

(c) If A and B are Hermitian matrices, show that $AB + BA$ is Hermitian whereas $AB - BA$ is skew-Hermitian.

(d) Verify that $(AB)^T = B^T A^T$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$$

(e) Given matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

show that $\sigma_1\sigma_2 - \sigma_2\sigma_1 = 2i\sigma_3$.

(f) Using the property of gamma function evaluate the integral

$$\int_0^\infty x^4 e^{-x}\, dx$$

Contd.

(g) Write the degree and order of the following partial differential equations :

(i) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$

(ii) $\left(\dfrac{\partial u}{\partial x}\right)^3 + \dfrac{\partial u}{\partial t} = 0$

(h) Find the value of a_0 of the Fourier series for the function $f(x) = x\cos x$ in the interval $-\pi < x < \pi$.

3. Answer **any three** of the following questions : $5 \times 3 = 15$

(a) (i) Why is the function $\left(1 - 2xh + h^2\right)^{-\frac{1}{2}}$ known as a generating function of Legendre polynomial ? 1

(ii) Show that

$$\left(1 - 2xh + h^2\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} P_n(x) h^n$$

where $P_n(x)$ is the Legendre polynomial. 4

(b) Evaluate explicitly the Legendre's polynomials $P_2(x)$ and $P_3(x)$. $2\frac{1}{2} + 2\frac{1}{2} = 5$

(c) Write the recursion formula for gamma function. Prove that

$$\Gamma\left(\dfrac{1}{2}\right) = \sqrt{\pi} = 1 \cdot 772$$

(d) What is diagonalize matrix ? Diagonalize the following matrix : $1 + 4 = 5$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(e) Express the matrix :

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$ as a sum of symmetric and skew-symmetric matrix.

(f) What is adjoint of a matrix ? For the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ verify the theorem

$$A \cdot (Adj A) = (Adj A) \cdot A = |A| \cdot I$$

where I is unit matrix. $1 + 4 = 5$

(g) If the solution $y(x)$ of Hermite's differential equation is written as

$$y(x) = \sum_{r=0}^{\infty} a_r \, x^{k+r}, \text{ show that the allowed}$$

values of k are zero and one only.

(h) Find the Fourier series representing

$$f(x) = x, \quad 0 < x < 2\pi$$

4. Answer **any three** of the following questions :
$$10 \times 3 = 30$$

(a) (i) Verify that the matrix

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix} \text{ is orthogonal.} \quad 2$$

(ii) Verify Cayley-Hamilton theorem for

the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and also

find A^{-1}. $\qquad$ 5+3=8

(b) Obtain the power series solution of the Legendre equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

(c) (i) Obtain the following orthogonality property of Legendre polynomial :

$$\int_{-1}^{+1} P_n(x)P_m(x)dx = 0 \text{ for } m \neq n \quad 6$$

(ii) Show that
$$H_0(x) = 1 \text{ and } H_1(x) = 2x \quad 2+2=4$$

(d) Prove the following recurrence relations : $\qquad$ 4+3+3=10

(i) $n P_n = (2n-1)x \, P_{n-1} - (n-1)P_{n-2}$

(ii) $x P_n' - P_{n-1}' = n P_n$

(iii) $2x H_n(x) = 2n H_{n-1}(x) + H_{n+1}(x)$

(e) What is periodic function ? Express the periodic functions in a series of sine and cosine functions. What are Fourier coefficients ? Determine the Fourier coefficients. $\qquad$ 1+1+1+7=10

(f) (i) Using the method of separation of variables, solve : $\qquad$ 6

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ where } u(x,0) = 6e^{-3x}$$

(ii) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \qquad 4$$

(g) *(i)* If $H_n(x)$ be the polynomial of Hermite differential equation, prove that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n^2(x)\, dx = 2^n \sqrt{\pi}.\, n! \qquad 7$$

(ii) Prove that the following matrix is unitary :

$$\begin{bmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\[2mm] \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{bmatrix} \qquad 3$$

(h) Deduce the one dimensional wave equation of transversely vibrating string under tension T. Solve the equation by the method of separation of variables.

7+3=10

3 (Sem – 3 / CBCS) PHY HC 2

2022

PHYSICS

(Honours)

Paper : PHY–HC– 3026

(Thermal Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any seven** of the following questions : 1×7=7

   (a) What is a cyclic process ?

   (b) Which state of matter has the highest entropy ?

   (c) How does root mean square velocity change with temperature ?

   (d) What is velocity space ?

(e) Name the transport phenomenon present in a gas that involves transfer of energy.

(f) Write the S.I. unit of Van der Waals' constant 'b'.

(g) Why does the pressure of a gas in a container wall increase when it is heated ?

(h) Is a 'closed system' an 'isolated system' ?

(i) How does the viscosity of a gas vary with pressure ?

(j) Can Gibbs' free energy be negative ?

(k) What is the origin of Doppler broadening in spectral lines ?

(l) In Brownian motion, how does size of the particle affect the speed of the particle ?

2. Answer **any four** of the following questions :

2×4=8

(a) At what temperature will root mean square velocity of a gas be half its value at $0°C$.

(b) Represent isobaric process in a P–V diagram.

(c) Evaluate Boyle temperature of a gas if its critical temperature is $5.5K$.

(d) Consider a system at room temperature. Explain about the value of entropy for the following situations :

   (i) temperature of the system is increased and reached equilibrium state

   (ii) temperature is decreased to $0K$.

(e) Explain physical significance of zeroth law of thermodynamics.

(f) How mean free path of a molecule is affected by temperature ?

(g) Why does the area of the Maxwell-Boltzmann velocity distribution curve always remain equal to unity ? Explain.

(h) Why specific heat of a gas at constant pressure is always greater than the specific heat of a gas at constant volume ?

3. Answer **any three** of the following questions : 5×3=15

(a) Find the change in entropy of the universe as a result of the following processes : 2½+2½=5

(i) A copper block of $400gm$ mass and with thermal capacity (at constant pressure) of $150J/deg$ at $100°C$ is placed in a lake at $10°C$.

(ii) The same block at $10°C$ is dropped from a height of $100m$ into the lake.

(b) What are the *four* thermodynamic potentials ? How specific heat at constant pressure can be expressed in terms of enthalpy ? 4+1=5

(c) Find an expression for coefficient of performance of a refrigerator.

(d) Derive $C_P - C_V = R$ for perfect gas from Maxwell's thermodynamic relations.

(e) Calculate the average speed and most probable speed of 1 mole of hydrogen molecule at $300K$. Neglect the mass of electron. 2½+2½=5

(f) Derive an expression for work done during an isothermal process.

(g) A Carnot engine absorbs $100J$ of heat from a reservoir at a temperature of the normal boiling point of water and rejects heat to a reservoir at the temperature of triple point of water. Find the heat rejected by the engine and its thermal efficiency. 2½+2½=5

(h) Show that at the critical temperature, the departure of Van der Waals' gas law from perfect gas law measures 62.5%.

4. Answer **any three** of the following questions : 10×3=30

(a) State Carnot's theorem. Briefly state the operations of a Carnot cycle by plotting them in (i) P–V diagram and (ii) T–S diagram. Show from T–S diagram that the efficiency of the cycle is $1 - \dfrac{T_2}{T_1}$, being independent of the nature of the working substance, where T_1 and T_2 are the source and sink temperature respectively. 2+3+3+2=10

(b) Derive all three TdS equations. Write physical significance of TdS equations. 3+3+3+1=10

(c) What is Joule-Thomson effect ? Derive an expression for Joule-Thomson coefficient. Find the values of Joule-Thomson coefficient for a perfect gas and a real gas. 2+3+2+3=10

(d) Derive Maxwell-Boltzmann's velocity distribution law.

(e) What are critical constants of a gas ? Obtain their values in terms of the constants of Van der Waals' equation. Hence deduce the law of corresponding states. 3+3+4=10

(f) Define coefficient of thermal conductivity. Show that coefficient of thermal conductivity $K = \eta C_V$ for an ideal gas, where η is coefficient of viscosity and C_V is specific heat at constant volume.

(g) Define free path and mean free path. What do you mean by 'collision probability' ? Show that the probability of a gas molecule traversing a distance x without collision is $e^{-x/\lambda}$ where λ is the mean free path of the gas molecules. 1+1+2+6=10

(h) Write short notes on the following : *(any two)* 5×2=10

(i) Unattainability of absolute zero

(ii) Adiabatic demagnetization

(iii) Andrew's experiment of CO_2 gas

(iv) Brownian Motion

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3 (Sem–3/CBCS) PHY HC 3

2022

PHYSICS

(Honours)

Paper : PHY–HC–3036

(Digital Systems and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :
   (any seven) 1×7=7

   (i) The deflection sensitivity of a CRO can be enhanced by reducing _____.

   (Fill in the blank)

(ii) The intel 8085 microprocessor is a 16 bits processor.

(State True or False)
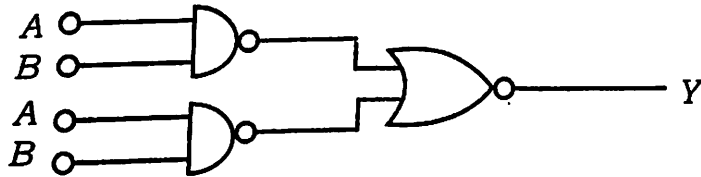
(iii) The design of flip-flops are based on

   (a) Sequential logic

   (b) Multiplexing

   (c) Combinational logic

   (d) Demultiplexing

   (Choose the correct option)

(iv) The full form of MDR is _____.

(Fill in the blank)

(v) For the given circuit diagram, the output Y is

   (a) $A + B$

   (b) AB

   (c) $\overline{A + B}$

   (d) $\overline{A.B}$

   (Choose the correct option)

(vi) The storage capacity of each stage in a shift register is _____ bits.

(Fill in the blank)

(vii) Monostable multivibrators can be used as frequency divider by using

   (a) Sawtooth wave generator

   (b) Triangular wave generator

   (c) Sine wave generator

   (d) Square wave generator

   (Choose the correct option)

(viii) What is the full form of VLSI ?

(ix) Mention *one* advantage of a digital circuit over an analog circuit.

(x) Write the names of an active component and a passive component in a circuit.

(xi) Convert the binary number 101.11_2 into decimal number.

(xii) Convert the decimal number 54.50 into binary number.

2. Answer the following questions in brief : **(any four)**　2×4=8

   (i) Convert the following hexadecimal numbers to binary

   (a) B32

   (b) AE2·4

   (ii) Reduce the following Boolean function

   $$A\overline{B} + \overline{A}B + AB + \overline{A}\overline{B}$$

   (iii) Using 2's complement, perform the subtraction

   101·1101 – 101·0111

   (iv) What do you mean by D/A converter ? Name *two* types of D/A converter.

   (v) What is synchronous counter ? Write *two* basic applications of counters.

   (vi) What is Primary memory ? What is its function ?

   (vii) Draw the logic symbol of XOR gate and construct the truth table.

   (viii) Mention *two* applications of Multiplexers.

3. Answer **any three** questions from the following :　5×3=15

   (i) (a) Convert the following Boolean expression into standard SOP form

   $$A + \overline{B} + CA$$

   (b) Show that $(\overline{AC} + B)(\overline{\overline{A} + \overline{C}}) = BAC$

   (ii) Explain how SR flip-flop is obtained from using NAND gates. Draw the truth table.

   (iii) With neat diagram explain the working of a serial-in parallel-out shift register. What is the basic difference between a shift register and a counter ?

   (iv) Simplify the Boolean function

   $$F(A, B, C, D) = \sum(0,1,2,4,5,6,8,9,12,13,14)$$

   with the help of K-maps.

   (v) Write short notes on : **(any one)**

   (a) BCD decade counter

   (b) Astable Multivibrator

   (c) Clocked D flip-flop

   (vi) Define Opcode and Operand. Write an 8085 Assembly Language Program (ALP) to store data of register C into memory location 2054H.　2+3=5

(vii) Prove the following equations with the help of truth tables :

(a) $\overline{A+B} = \overline{A}.\overline{B}$

(b) $\overline{A.B} = \overline{A}+\overline{B}$

(viii) Using NAND gates only, realize the following gates :

(a) AND

(b) OR

(c) NOT

4. Answer **any three** of the following questions:
$$10 \times 3 = 30$$

(i) What are decoders and encoders ? With the help of a logic diagram and truth table explain. 2+4+4=10

(a) 2 to 4 decoder

(b) Octal to binary encoder.

(ii) What is a full Adder ? Draw the block diagram and truth table of a full Adder. Design a full Adder logic circuit by applying Karnaugh map. 1+4+5=10

(iii) (a) Draw the K-map to minimize the following expression.

$$\overline{ABC} + A\overline{BC} + \overline{A}B\overline{C} + AB\overline{C}$$

(b) For the given truth table, find the minimized logical expression by the use of K-map and SOP method and draw the equivalent logic circuit : 4+6=10

A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(iv) With the help of a logic diagram and function table explain 5×2=10

(a) 1 to 2 Demultiplexer

(b) 2 to 1 Multiplexer.

(v) Draw the block diagram of a CRO. What is the function of electron gun in a CRO ? Explain how the phase difference between two sinusoidal voltages of the same frequency and same amplitude can be determined by using CRO.

3+2+5=10

(vii) Prove the following equations with the help of truth tables :

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A	B	C	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(iv) With the help of a logic diagram and function table explain $5 \times 2 = 10$

(a) 1 to 2 Demultiplexer

(b) 2 to 1 Multiplexer.

(v) Draw the block diagram of a CRO. What is the function of electron gun in a CRO ? Explain how the phase difference between two sinusoidal voltages of the same frequency and same amplitude can be determined by using CRO.

$3+2+5=10$

Contd.

(vi) What is a microprocessor ? What is the difference between microprocessor and microcomputer ? Write the basic features of Intel 8085 microprocessor.

1+1+8=10

(vii) (a) Give the symbol and truth table of XNOR gate.

(b) Realize the NOT gate using transistor.

(c) What are preset and clear operations ? 3+3+4=10

(viii) (a) What do you mean by the following terms used in a Microprocessor ?
(i) Buses
(ii) Registers
(iii) ALU

(b) What is a Half Subtractor ? Give the symbol and the truth table.

6+4=10

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