Total number of printed pages–8

3 (Sem–3/CBCS) STA HC 1

2022

STATISTICS

(Honours)

Paper : STA–HC–3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : 1×7=7

(a) If X and Y are *two* independent chi square variates with n_1 and n_2 d.f. respectively, then $U = X/Y$ follows

(i) $\beta_2(n_1/2, n_2/2)$

(ii) $\beta_1(n_1, n_2)$

(iii) F distribution

(iv) None of the above

(Choose the correct option)

Contd.

(b) The skewness in a chi-square distribution will be zero, if

(i) $n \to \infty$

(ii) $n = 0$

(iii) $n = 1$

(iv) $n < 0$

(Choose the correct option)

(c) Level of significance is the probability of

(i) type I error

(ii) type II error

(iii) not committing error

(iv) Any of the above

(Choose the correct option)

(d) Student's *t*-test is applicable in case of

(i) small samples

(ii) samples of size between 5 and 30

(iii) large samples

(iv) None of the above

(Choose the correct option)

(e) Equality of *two* population variances can be tested by

(i) t-test

(ii) F-test

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(Choose the correct option)

(f) Test of hypothesis $H_0 : \mu = 70$ vs $H_1 : \mu > 70$ leads to

(i) one sided left tailed test

(ii) one sided right tailed test

(iii) two tailed test

(iv) None of the above

(Choose the correct option)

(g) Degress of freedom for statistic – χ^2 in case of contingency table of order (2×2) is

(i) 3

(ii) 4

(iii) 2

(iv) 1

(Choose the correct option)

(h) Analysis of variance utilises

(i) F-test

(ii) χ^2-test

(iii) Z-test

(iv) t-test

(Choose the correct option)

(i) If β is the probability of type II error, the power of the test is ______. *(Fill in the blank)*

(j) Critical region is also known as ______. *(Fill in the blank)*

(k) t-distribution with 1 d. f. reduces to ______. *(Fill in the blank)*

(l) The value of chi square varies from ______ to ______. *(Fill in the blanks)*

2. Answer ***any four*** questions of the following : 2×4=8

(a) A random sample of size 4 is drawn from the discrete uniform distribution

$$P(X = x) = \frac{1}{6};\ x = 1, 2, 3, 4, 5, 6$$

Obtain the distribution of the smallest and largest order statistic.

(b) Obtain the moment generating function of chi square distribution.

(c) Define sampling distribution and standard error.

(d) State the applications of order statistics.

(e) Under what conditions is χ^2 test valid ?

(f) State the important applications of F distributions.

(g) Define critical region and level of significance.

(h) State the applications of t-distribution.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Derive Fisher's t-distribution.

(b) For 2×2 contingency table

a	b
c	d

,

prove that chi square test of independence given

$$\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)},\ N = a+b+c+d.$$

(c) Show that the mgf of $y = \log \chi^2$, where χ^2 follows chi square distribution with n df is

$$M_Y(t) = 2^t \left.\Gamma\left(\frac{n}{2}+t\right)\right/ \Gamma(n/2)$$

(d) Show that for t-distribution with n df the mean deviation about mean is

$$\sqrt{n}\,\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{n}{2}\right)$$

(e) There are two populations and P_1 and P_2 are the proportions of members in the two populations belonging to 'low income' group. It is desired to test the hypothesis $H_0 : P_1 = P_2$. Explain clearly, the procedure that you would follow to carry out the above test at 5% level of significance.

(f) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0,1)$. Define $\overline{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i$ and

$$\overline{X}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i$$

Find the distribution of

(a) $\frac{1}{2}\left(\overline{X}_k + \overline{X}_{n-k}\right)$

(b) $k\overline{X}_k^2 + (n-k)\overline{X}_{n-k}^2$

(g) Show how probability points of $F(n_2, n_1)$ can be obtained from those of $F(n_1, n_2)$.

(h) For the exponential distribution $f(x) = e^{-x}, x \geq 0$; show that the c.d.f. of $X(n)$ in a random sample of size n is $F_n(X) = \left(1 - e^{-x}\right)^n$.

4. Answer ***any three*** of the following questions : 10×3=30

(a) If X_1 and X_2 are two independent χ^2-variates with n_1 and n_2 df respectively, then show that X_1/X_2 is a $\beta_2(n_1/2, n_2/2)$ variate.

(b) Find the variance of the t-distribution with n df $(n > 2)$.

(c) Derive the relation between F and χ^2 distribution.

(d) Derive Snedecor's F distribution.

(e) X is a binomial variate with parameters n and p and f_{v_1,v_2} is an F-statistic with v_1 and v_2 df. Prove that

$$P(X \le k-1) = P\left[f_{2k}, 2(n-k+1) > \left(\frac{n-k+1}{k}\right)\left(\frac{p}{1-p}\right)\right]$$

(f) If X_1 and X_2 are independent χ^2-variates with n_1 and n_2 df respectively and $U = X_1/(X_1+X_2)$ and $V = X_1 + X_2$ are independently distributed, show that U is a $\beta_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate and V is a χ^2 variate with (n_1+n_2) df.

(g) Express the constants y_0, a and m of the distribution

$$dF(x) = y_0\left(1 - x^2/a^2\right)^m dx, -a < x < a$$

in terms of its μ_2 and β_2.

(h) If the random variables X_1 and X_2 are independent and follow chi-square distribution with n df, show that $\sqrt{n}(X_1 - X_2)/2\sqrt{X_1 X_2}$ is distributed as student's t with df independently of $(X_1 + X_2)$.

Total number of printed pages–7

3 (Sem–3/CBCS) STA HC 2

2022

STATISTICS

(Honours)

Paper : STA–HC–3026

(Sampling and Indian Official Statistics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** : 1×7=7

(a) Finite population correction factor is

(i) $\frac{n}{N}$

(ii) $1-\frac{n}{N}$

(iii) $1+\frac{n}{N}$

(iv) None of the above

(Choose the correct option)

Contd.

(b) Judgement sampling is

(i) probabilistic

(ii) non-probabilistic

(iii) mixed

(Choose the correct option)

(c) Simple random sampling is also known as ______ random sampling. *(Fill in the blank)*

(d) A complete list of sampling units which represents the population to be covered is called the ______. *(Fill in the blank)*

(e) Sub-sampling is also known as two-stage sampling. *(State True **or** False)*

(f) Systematic sampling is more precise than simple random sampling only if units within the sample are ______. *(Fill in the blank)*

(g) State the condition under which the regression estimator reduces to the ratio estimator.

(h) Name different series of random sampling numbers.

(i) In two-stage sampling, what sampling design is used to select second-stage units from the selected first-stage units?

(j) Name *any two* principles to be followed in stratifying a population.

(k) Name the sampling procedure where the probability of selection is proportional to the size of the unit.

(l) If all the units of a population are surveyed, it is called ______. *(Fill in the blank)*

2. Answer ***any four*** : 2×4=8

(a) Show that in stratified random sampling, with usual notations

$$V(\bar{y}_{st}) = \frac{1}{N^2}\sum_{i=1}^{k} N_i (N_i - n_i)\frac{S_i^2}{n_i}$$

(b) Obtain an unbiased estimate of the population in case of systematic sampling, when the population consist of $N = nk$ units.

(c) Define two-stage sampling.

(d) What is the basic difference between simple random sampling and P.P.S sampling procedures?

(b) Judgement sampling is

(i) probabilistic

(ii) non-probabilistic

(iii) mixed

(Choose the correct option)

(c) Simple random sampling is also known as ______ random sampling.

(Fill in the blank)

(d) A complete list of sampling units which represents the population to be covered is called the ______. *(Fill in the blank)*

(e) Sub-sampling is also known as two-stage sampling. *(State True **or** False)*

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(g) State the condition under which the regression estimator reduces to the ratio estimator.

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(j) Name *any two* principles to be followed in stratifying a population.

(k) Name the sampling procedure where the probability of selection is proportional to the size of the unit.

(l) If all the units of a population are surveyed, it is called ______.

(Fill in the blank)

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(a) Show that in stratified random sampling, with usual notations

$$V(\bar{y}_{st}) = \frac{1}{N^2}\sum_{i=1}^{k} N_i(N_i - n_i)\frac{S_i^2}{n_i}$$

(b) Obtain an unbiased estimate of the population in case of systematic sampling, when the population consist of $N = nk$ units.

(c) Define two-stage sampling.

(d) What is the basic difference between simple random sampling and P.P.S sampling procedures ?

(e) How does sample survey differ from complete census ?

(f) When does one go for stratification in sample surveys ?

(g) Define sampling unit and sampling frame.

(h) Define sampling and non-sampling errors in sample survey.

3. Answer ***any three*** : 5×3=15

(a) With usual notations, show that in stratified random sampling variance of $\overline{y}_{st}$ is minimum for fixed total size of the sample.

(b) Discuss the basic principles of a sample survey.

(c) Show that in SRSWOR and SRSWR the sample mean is an unbiased estimate of the population mean, i.e., $E(\overline{y}_n) = \overline{Y}_N$.

(d) What are the advantages of sample survey over complete enumeration (census) ?

(e) Show that the systematic sampling is more precise than the simple random sampling, if the variance within the systematic sampling is larger than the population mean square.

(f) In what situations the cluster sampling is preferred ? Comment on the efficiency of cluster sampling as compared to simple random sampling.

(g) In what situation systematic sampling is preferred over other sampling procedures ? Discuss the advantages and disadvantages of systematic sampling.

(h) Describe the method of collection of official statistics in India.

4. Answer ***any three*** of the following questions : 10×3=30

(a) Explain the principal steps involved in the planning and execution of a sample survey.

(b) Prove that in simple random sampling, the sample mean is the best linear unbiased estimate (blue) of the population mean. What do you mean by margin of errors in the estimate ?

(c) Carry out the comparison of simple random sampling, stratified random sampling and systematic sampling in the presence of linear trend.

(d) With usual notation, show that
$V_{rad} \geq V_{prop} \geq V_{opt}$

(e) Explain ratio estimator and regression estimator. When is regression estimator preferred over ratio estimator ?

(f) Write notes on :

(i) Origin and function of Central Statistical Orgination (CSO) and its publications

(ii) Directoriate General of Commercial and Intelligence Statisties (DGCIS) and its publications

(g) Write a note on the origin and function of National Sample Survey Office (NSSO).

(h) Write notes on :

(i) Indian Statistical Service

(ii) Role of Ministry of Statistics and Program Implementation (MoSPI).

(b) Prove that in simple random sampling, the sample mean is the best linear unbiased estimate (blue) of the population mean. What do you mean by margin of errors in the estimate ?

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(i) Indian Statistical Service

(ii) Role of Ministry of Statistics and Program Implementation (MoSPI).

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3 (Sem–3/CBCS) STA HC 3

2022

STATISTICS

(Honours)

Paper : STA–HC–3036

(Mathematical Analysis)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** from the following questions : $1 \times 7 = 7$

   (a) Identify the wrong statement :

      (i) The set R of real numbers is the neighbourhood of each of its points.

      (ii) The set Q of rationals is the neighbourhood of each of its points.

      (iii) The open interval $]a, b[$ is the neighbourhood of each of its points.

   (Choose the correct option)

(b) The set $\left\{\frac{1}{n} : n \in N\right\}$ has only one limit point, zero, which is not a member of the set. *(State True or False)*

(c) If $\sum u_n$ is a positive term series, such that $\lim_{n\to\infty} \frac{u_n + 1}{u_n} = l$, then the series converges if

(i) $l < 1$

(ii) $l > 1$

(iii) $l = 1$

(iv) $l = 0$ *(Choose the correct option)*

(d) The positive term geometric series $1 + r + r^2 + ...$ converges for $r < 1$ and diverges for $r \geq 1$.

(State True or False)

(e) Define alternating series.

(f) A function which is continuous in a closed interval is also uniformly continuous in that interval.

(State True or False)

(g) State the geometrical interpretation of Lagrange's mean value theorem.

(h) State the expansion of *cos x*.

(i) The *n*th divided difference can be expressed as the product of multiple integrals. *(State True or False)*

(j) Define the operators μ, δ used in calculus of finite differences.

(k) State Stirling's formula for factorial *n*, when *n* is large.

(l) Which of the following is not correct ?

(i) Simpson's rule gives a better result than the trapezoidal rule.

(ii) Weddle's rule is generally more accurate than any of the others.

(iii) Simpson's $1/3$ rule is better than Simpon's $3/8$ rule

(iv) None of the above

(b) The set $\left\{\frac{1}{n} : n \in N\right\}$ has only one limit point, zero, which is not a member of the set. *(State True or False)*

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(State True or False)

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(f) A function which is continuous in a closed interval is also uniformly continuous in that interval.

(State True or False)

(g) State the geometrical interpretation of Lagrange's mean value theorem.

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(iv) None of the above

2. Answer ***any four*** from the following questions : 2×4=8

(a) Show that the series

$\frac{1}{1^P}-\frac{1}{2^P}+\frac{1}{3^P}-\frac{1}{4^P}+\ldots\ldots$ converges for $P>0$.

(b) Define bounded and unbounded sets. Is the set of natural numbers bounded?

(c) If M and N are neighbourhood of a point x, then prove that $M \cap N$ is also a neighbourhood of x.

(d) State Taylor's theorem with Lagrange's and Cauchy's form of remainder.

(e) Using Lagrange's mean value theorem prove that

$|\tan^{-1} x - \tan^{-1} y| \leq |x-y| \; \forall x, y \in R$

(f) Solve the difference equation

$u_{x+1} - au_x = 0, a \neq 1$

(g) Write a note on numerical integration.

(h) Find the first three divided differences of the function $\frac{1}{x}$ for the arguments a, b, c, d.

3. Answer ***any three*** from the following questions : 5×3=15

(a) Show that $\lim_{n \to \infty} n^{\frac{1}{n}} = 1$

(b) Expand e^x by Maclaurin's infinite series.

(c) (i) Define limit superior and limit inferior of a bounded sequence. 2

(ii) Prove that the intersection of a finite family of open sets is open. 3

(d) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

(e) State the following :

(i) d'Alembert's ratio text 2

(ii) Raabe's test 2

(iii) Absolute convergence of series 1

(f) State and prove Simpson's $\frac{1}{3}$ rule.

(g) Establish the relation between operator E of calculus of finite differences and differential coefficient D of differential calculus.

Also show that

$$\nabla = 1 - e^{-hD}$$

where ∇ is called backward difference operation. 3+2=5

(h) State and prove Gauss's forward interpolation formula.

4. Answer ***any three*** of the following questions : 10×3=30

(a) State and prove Cauchy's general principle of convergence.

(b) Prove that if $f(x)$ is a function, which is

(i) continuous in the closed interval $[a, b]$

(ii) differentiable in the open interval (a, b) and

(iii) $f(a) = f(b)$, then there exists one value of x say $\xi \in]a, b[$ such that $f(\xi) = 0$

Also give the geometrical meaning of Rolle's theorem. 8+2=10

(c) Expand $log(1+x)$ by Maclaurin's infinite series.

(d) *(i)* State Cauchy's first theorem on limits. 1

(ii) Show that

$$\lim_{n \to \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right] = 1$$

6

(iii) Define *(i)* monotonic sequence and *(ii)* derived sets. 3

(e) *(i)* State and prove Lagrange's mean value theorem. Also give its geometrical interpretation. 5+2=7

(f) State and prove Simpson's $\frac{1}{3}$ rule.

(g) Establish the relation between operator E of calculus of finite differences and differential coefficient D of differential calculus.

Also show that

$$\nabla = 1 - e^{-hD}$$

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6

(iii) Define (i) monotonic sequence and (ii) derived sets. 3

(e) (i) State and prove Lagrange's mean value theorem. Also give its geometrical interpretation. 5+2=7

(ii) Show that a necessary condition for convergence of an infinite series $\sum u_n$ is that $\lim_{n \to \infty} \sum u_n = 0$. 3

(f) State and prove Lagrange's interpolation formula for unequal intervals. Also show that the sum of the Lagrangian coefficients is unity. 7+3=10

(g) (i) State and prove Simpson's $3/8$ rule. Also state its assumptions. 6

(ii) Solve the difference equation

$u_{x+1} - 3^x u_x = 0$ 4

(h) (i) Write a note on use of various interpolation formulae. 5

(ii) Evaluate

$$\Delta^2 \left[\frac{5x+12}{x^2+5x+6} \right], \text{ taking } h = 1$$ 5
